[illegible] [illegible] [illegible]. 3

[illegible]([illegible]): [illegible]

[illegible]: [illegible] [illegible] [illegible] [illegible] [illegible].

I)

[illegible] [illegible] [illegible] ∂.

$0, 1 \in O$

$* : P(O) \to O$

$\nu\omega \in O : 0 + \omega = \omega$

$x : P(O) \to O$

$\nu\omega \in O : 1.\omega = \omega$

$\nu\omega \in O : 0.\omega = 0$

$|| \; || : O \to O$

$|| 0 || = 0$

$|| 1 || = 1$

$Op = \{ ||\omega|| \mid \omega \in O \setminus \{0\} \}$, (p : positive-definite)

$Op = \partial(O \setminus \{0\})$

$\partial = \{ x(v) \mid v \in P(\partial) \}$

$\forall i \in \partial : \partial'(z') = \partial^2$, $(z \in O)$

$\partial^0 = 1$

$Op = \partial Kp,o = Kp,o$

$Ko = (\partial Kp,o) \cup \{0\}$, $(j \in \partial \setminus \{1\})$

$x : P(Ko) \to Ko$

$|| \; || : P(Ko) \to Ko$

$Ko = \{ x(k) \mid k \in P(Ko) \}$

□□□ □□□ □□ □□□ ∂□ □□ □□, □□ (□ □□), □□, □□, □□□□ □□, □□, □□ □ □ n-□ □□□ □□□□ □□□□□□.

$\Sigma(x) = \Sigma$ i

ied \ {z}

$\forall i \in \partial : \Sigma(i) + i = 0$

$\alpha \in O \setminus \{0\}, i \in \partial$

$\partial'(x) = \alpha^2 x + \Sigma(i)$

$\partial\Sigma(i)(x) = (\alpha + i)\Sigma(i)x + i$

□□□□ theta □□

$\exp(iz) = \cos z + i \sin z, (z \in C)$

$\kappa > 0$

$Vk := \{ \exp(i\theta) : \theta \in R, \exp(i\theta \cdot k) = 1 \}.$

$\Omega k := \{ vr : v \in Vk, r > 0 \} \cup \{ 0 \}$

$P : \Omega k \to R_{\geq 0}$

□ □□ k □ □□□(quasi-homogeneous) □□□□□. □, □□ $t \in N$ □ □□ $x = (x1, ..., xn) \in \Omega k$ □ □□ □□□ □□ □□□ □□ □□□ $w1, ..., wn > 0$ □ □□□□□.

$P(tw1x1, tw2x2, ..., twnxn) = tkP(x).$

$\Lambda k := \{ vr : v \in Vk, r \in N \} \cup \{ 0 \}$

$\theta P(\tau) := \Sigma \exp [- (P(x) dx) / (P(x)\tau)]$

$x \in \Lambda k$

□□ 1 (□□□ □□). k1, k2, ..., kn □ □□ □□ □□□□□ □□ $k > 0$ □ □□□□ □□□. C^1 □□ $P : \Omega \to (0, \infty)$ (□□□ $\Omega \subset Rn$ □ □□ □□)□ □□ $t > 0$ □ □□ $x = (x1, ..., xn) \in \Omega$ □ □□ □□□ □□ □□ □□□ (k1, ..., kn)□ □□ □□ k □ □□□ □□□□ □□□.

$P(tk1x1, tk2x2, ..., tknxn) = tkP(x1, x2, ..., xn).$

□, i □□ □□□ tki □ □□□□□□ □□□ □□ tk □ □□□□□□□. □□□ P(x)□ Ω □ □□□ □□□ □ +∞□ □□□□ (□□ □□□ □□□□□ □□) □□ $x \in \Omega$ □ □□ $\nabla P(x) \neq 0$ □□□ □□□□□ (□□□ P □ □□ □□□ □□□□ □□□□□□). □□□□□ □□ □□□ $t = 1$ □□ □□ □□□□ □□□□ □□ □□ □□□□ □□□□□□.

$k1x1\ \partial P/\partial x1(x) + k2x2\ \partial P/\partial x2(x) + ... + knxn\ \partial P/\partial xn(x) = kP(x).$ (1)

□□□ □□ $P(x) = 1$ □ □□ □□ (□□)□ ∑□ □□□□□.

$\sum := \{ x \in \Omega : P(x) = 1 \}.$

Ω □□ $\nabla P \neq 0$ □□□ □□ □ □□□ □□□□ (n - 1)□□ □□□□□. □□ □□ d-1 □ ∑□ (n - 1)□□ “□□ □□ □□” □ □□□□□.

$V = \int_{\sum} 1/||\nabla P(x)||\ dS(x),$

□□□ $dS(x)$□ $\sum$□ □□□□ □□□ □□□□ $\|\nabla P(x)\|$□ □□□□ □□□□ □□□□□ (□ □□□□ □□ □□□□ □□□□□ □□□□□). □□□ □□ □□□ □□ V□ □□□□□ (□□, $\sum$□ $P(x) \to \infty$□□□ Ω□□ □□□□□□).

□□ □□:

□□ 1 ($e^{-P(x)}$□ □□). □ □□ □□□ Ω□ □□ $e^{-P(x)}$□ □□□ □□ □□□ □□□ □ □□□□. □□, □□□ □□□□□.

$$\int_\Omega e^{-P(x)}\, dx = \Gamma((k_1 + \dots + k_n)/k) \int_{P(x)=1} 1/\|\nabla P(x)\|\, dS(x),$$

□□□ □□□ □□ □□□ □□ □□ $\{P(x) = 1\}$□ □□ □□ □□□□ □□□□□. □□□□,

$$\int_\Omega e^{-P(x)}\, dx = \Gamma((k_1 + \dots + k_n)/k)\, V.$$

□□. 1 □□: □□ □□. P□ □□□□□ □□□ □□ □□□ □□□□□. $P(x) > 0$□ □ x□ □□ □□□ □□□□□.

$$r := P(x)^{1/k} \;□\; \xi := (x_1/r^{k_1}, \dots, x_n/r^{k_n}).$$

r□ □□□ □□ $r^k = P(x)$□□□ □□□ □□□ □□□□ □□□ □□□□.

$$P(\xi) = P(x_1/r^{k_1}, \dots, x_n/r^{k_n}) = 1/r^k\, P(x) = 1.$$

□□□ $\xi \in \Sigma = \{P(x) = 1\}$□□□. □, □□□ $x \in \Omega$□ $r > 0$, $\xi = (\xi_1, \dots, \xi_n) \in \Sigma$□ □□□ □□□□ □□□ □ □□□□.

$$x = r^k\xi, \quad (2)$$

(□ □□□ □ $x_i > 0$□ □□□□ x□ □□□□ □ □□□□□□. □□ □□□□ □□ □□ □□□ □ □□□□. $P(x)$□ □□□□ □□ □□□□ □□ $x_i > 0$□□ □□□□ □□□□□. □ □□□□□ □□□ □□□ □□□ □□□□ □□□□ □ □□ □□□ □□□ □ □□□□.)

(2)□ □□□□ $\phi: (r, \xi) \to x$ □□□ $(0, \infty) \times \Sigma$□□ Ω□□ C^1 □□ □□ □□□□□. □ □□□ □□□□ □□□ □□□□□. □□ ϕ□ □□□ □□□ $J(r, \xi) := \det(\partial x/\partial(r, \xi))$□ □□□□□.

2 □□: □□□ □□□ □□. □□□ □ $(r, \xi) \in (0, \infty) \times \Sigma$□ □□□□□. $D\phi(r, \xi)$□ □□□□ □□ □□ □□ dr□ $d\xi = (d\xi_1, \dots, d\xi_n)$□ □□□□□. □□□ $d\xi$□ ξ□□ Σ□ □□□□ $d\xi$□ $\nabla P(\xi) \cdot d\xi = 0$□ □□□□□ ($\Sigma$□ □□ □□ $P(\xi) = 1$□ □□□□ □□□□□). (2)□□ $x_i = r^{k_i} \xi_i$□ □□□ □□□ □□□□.

$$dx = k_i r^{k_i-1} \xi_i\, dr + r^{k_i} d\xi_i.$$

□□ □□□□ □□ □□□ $D\phi(r, \xi): T_{(r, \xi)}((0, \infty) \times \Sigma) \to T(\mathbb{R}^n)$□ i□□ □□□□□.

$$\partial x_i/\partial r = k_i r^{k_i-1} \xi_i, \quad \partial x_i/\partial \xi_j = r^{k_i} \partial \xi_i/\partial \xi_j$$

$1 \le i \le n$□ $1 \le j \le n - 1$□ □□. $J(r, \xi) = |\det D\phi(r, \xi)|$□ □□□□ □□ □ □□□ □□□□ □□ □□□□□.

* r□□ □□□□ □□ □□: □ □□□ r□ □□ ξ □□□ □□□□□ □□□□□ ($\xi = \xi(\theta)$□ □□□ $(\theta_1, \dots, \theta_{n-1})$□ Σ□ □□ □□□□□□ □□□□□). □□□ □□□ □□□□□ □□□ □□ □□□ □ □□□□.

$$D\phi(r, \xi) = (k_i r^{k_i-1} \xi_i \;\; r^{k_1} \partial \xi_i/\partial \xi_1 \; \dots \; r^{k_n} \partial \xi_i/\partial \xi_n)$$

□ □□□ i□□ □□□ r^{k_i-1}□ □□ □□□□ $\prod_i r^{k_i-1} = r^{\sum_i k_i - n}$ □□□ □□□□□. □□□□ □ $\partial \xi_j$ □□□ r□ □□ □□□ □ □□□□ (□ $\partial \xi_j$ □□□ □ □□□ r^{k_i-1}□ □□□□□□ □ □□□ □□ □□ r□ □□□ □ □ □□□□ r□ □□□□□□ □□ □□ □□ r□ □□□ □□□□□.) □□□ □□□□□ r^{n-1}□ □□ □□□ □□□□. □□□ □□□ □□□□.

$J(r, \xi) = r^{(\sum_i k_i - n)} * r^{(n-1)} * |\det M(\xi)| = r^{(\sum_i k_i - 1)} * |\det M(\xi)|$, (3)

□□□ $M(\xi)$□ $n \times n$ □□(r□ □□□)□□□.

$M(\xi) := (k_1\xi_1 \ \partial\xi_1/\partial\theta_1 \ \ldots \ \partial\xi_1/\partial\theta_{n-1}$

$k_2\xi_2 \ \partial\xi_2/\partial\theta_1 \ \ldots \ \partial\xi_2/\partial\theta_{n-1}$

...

$k_n\xi_n \ \partial\xi_n/\partial\theta_1 \ \ldots \ \partial\xi_n/\partial\theta_{n-1})$

□□□ $M(\xi)$□ □ □□ □□ $(k_1\xi_1, \ldots, k_n\xi_n)$□□ □□□ □□ $j = 1, \ldots, n-1$□ □□ □□ □□ $\partial\xi/\partial\theta_j = (\partial\xi_1/\partial\theta_j, \ldots, \partial\xi_n/\partial\theta_j)^T$ □□□.

* Σ□ □□□□ □□□□ $\det M(\xi)$ □□: $M(\xi)$□ 2□□□□ n□□ □□ □□ □□ $T_\xi\Sigma$□ □□□□. □ □□ □ $\kappa_\xi := (k_1\xi_1, \ldots, k_n\xi_n)$□ □□□□□ Σ□ □□□ □□□□. □□ □□ □□□ □□□ □□□□. □□□ □□ □□□ □□□ (1)□ □□ □□□ □□□□□.

$k_1\xi_1 \, \partial P/\partial x_1(\xi) + \ldots + k_n\xi_n \, \partial P/\partial x_n(\xi) = kP(\xi) = k$,

$P(\xi) = 1$ on Σ □□□. □□ □□ κ_ξ□ □□ □□ $\nabla P(\xi)$□ 0□ □□ □□□ □□□□ □□ □□□□□.

$\nabla P(\xi) \cdot (k_1\xi_1, \ldots, k_n\xi_n) = k$. (4)

□□□ □ □□ □□ $\partial\xi/\partial\theta_j$□ $\nabla P(\xi)$□ □□□□□ ($\nabla P(\xi)$□ $T_\xi\Sigma$□ □□□□□). □□□ □ □□ □ κ_ξ□ □□ □ □□ □□□□ □□□ □ □□□□.

$\kappa_\xi = (\nabla P(\xi)/\|\nabla P(\xi)\|^2) * (k/\|\nabla P(\xi)\|) + (\kappa_\xi - (\nabla P(\xi)/\|\nabla P(\xi)\|^2) * (k/\|\nabla P(\xi)\|))$.

□□ □□ □□ □□

□□□□ □ □□ □□ $T_\xi\Sigma$□ □□□□ $M(\xi)$□ □□ □□ □□ □□□□□. □□□ □ □□ □□ □ □□ □□□ □□□ □□□□ (□□□□ □□□□ □□ □ □□) □ □ □□ □□ $(k/\|\nabla P(\xi)\|^2) * \nabla P(\xi)$□ □□□□. □□□□□ □□□ □□□□.

$\det M(\xi) = \det((k/\|\nabla P(\xi)\|^2) * \nabla P(\xi) \ \partial\xi/\partial\theta_1 \ \ldots \ \partial\xi/\partial\theta_{n-1})$,

□□□ □□□ □□□ □□ □□ $\nabla P(\xi)$□ □ □□ □□ □□ □□ □□□ □□□ □□ □□□□. □ □□□□ □□□□ $\{\nabla P(\xi), \partial\xi/\partial\theta_1, \ldots, \partial\xi/\partial\theta_{n-1}\}$□ □□ □□□ □□□□□□ n□□ □□□□□. $\nabla P(\xi)$□ □ $\partial\xi/\partial\theta_j$□ □□□□□ □ □□□ $\|\nabla P(\xi)\|$□ $\partial\xi/\partial\theta_1, \ldots, \partial\xi/\partial\theta_{n-1}$□ □□ □□□ □□□□□□ $(n-1)$□□ □□□ □□□ □□□□□□□. □□ □□□ □□□ □□□□.

$|\det(\nabla P(\xi) \ \partial\xi/\partial\theta_1 \ \ldots \ \partial\xi/\partial\theta_{n-1})| = \|\nabla P(\xi)\| * \|\partial\xi/\partial\theta_1 \wedge \ldots \wedge \partial\xi/\partial\theta_{n-1}\| = \|\nabla P(\xi)\| \, dS(\xi)$,

□□□ $dS(\xi)$□ $\xi \in \Sigma$ □□□ □□□ □□□□□. □□□ □□□ □□□□.

$\det M(\xi) = (k/\|\nabla P(\xi)\|^2) * \|\nabla P(\xi)\| \, dS(\xi) = (k/\|\nabla P(\xi)\|) \, dS(\xi)$.

□□ (3)□ □□□□ □ □□□□ □□□ □□□□ □□□ □□□ □□□ □□ □ □□□□.

$J(r, \xi) = r^{(\sum_i k_i - 1)} * (k/\|\nabla P(\xi)\|) \, dS(\xi)$. (5)

□ □□□ □□□□□ □□□□ “□□□” □□ □□□ □□□ □ □□□□. $r^{(\sum_i k_i - 1)} \, dr$ □ □□□ □□□ □□□ □□□ ($\sum_i k_i$ □ □ □□□□□□ $\mathbb{R}^n$□ □□ □□□□□). □□□ $\|\nabla P(\xi)\| \, dS(\xi)$□ □□□□ □□□□□ □□□□ $r^{(n-1)} \, dr \, d\Omega$ □ □□□□ □ □□(□□ Σ)□ □□□ □□□□□.

3 □□: □□ □ □□ □□ □□. □□ □□□ □□ □□ (2)□ □□□□□. □□ (5)□ □□ □□ □□ □□□ □□ $f(x)$□ □□ □□□ □□□□□.

$$\int_\Omega f(x)\,dx = \int_0^\infty \int_\Sigma f(r^{k_1}\xi_1, \dots, r^{k_n}\xi_n)\, J(r, \xi)\, dr\, d\sigma(\xi).$$

□□□ $d\sigma(\xi)$□ □□ Σ □ □□ □□□ □□□□□ (□, $d\sigma(\xi) = dS(\xi)/\|\nabla P(\xi)\|$). □□ □□ $f(x) = e^{-P(x)}$□ □□□□□. $P(r^{k_1}\xi_1, \dots, r^{k_n}\xi_n) = r^k P(\xi) = r^k$ (Σ □□ $P(\xi) = 1$ □□ □□)□ □□□□ □□□ □□□□.

$$\int_\Omega e^{-P(x)}\,dx = \int_0^\infty \int_\Sigma e^{-r^k}\, r^{\sum_i k_i - 1}\, (k/\|\nabla P(\xi)\|)\, dS(\xi)\, dr.$$

□□□ □□□□ (5)□ □□ $d\sigma(\xi)$□ □□□□□ $dS(\xi)/\|\nabla P(\xi)\|$□ □□ □□□□ □□□ □□□□($r^{k_1}, \dots, r^{k_n}$)□ □□□□ □□ □□ r^k □ □□□□□□. □□□ □□ $e^{-r^k} r^{\sum_i k_i - 1}$□ □□□ □□□□ □□□ □□(□□□ □□)□ □□ □□ □□□ □□ □ □□□□. □□□ □□□ □□□□.

$$\int_\Omega e^{-P(x)}\,dx = \int_\Sigma dS(\xi)/\|\nabla P(\xi)\| \int_0^\infty e^{-r^k}\, r^{\sum_i k_i - 1}\, dr. \quad (6)$$

□ □□ □□□ $V\kappa$ □ □□□ □□ □□□ □□□□.

$$\int_\Sigma (k/\|\nabla P(\xi)\|)\, dS(\xi) = k \int_{P(x)=1} 1/\|\nabla P(x)\|\, dS(x) = \kappa V.$$

(κ □ □□□□□ □□ □□□□ □□ □ □□□□.) (6)□ □ □□ □□□ □□ $r > 0$ □ □□□□ □□ $u = r^k$ □ □□ □□□ □□□ □ □□□□. □□□ $du = k\, r^{k-1}\, dr$ □□□ $r^{k-1}\, dr = 1/k\, du$ □□□. r □ 0 □□ ∞□ □□□ □ u □ □□□□□□□. □□□ □□□ □□□□.

$$\int_0^\infty e^{-r^k}\, r^{\sum_i k_i - 1}\, dr = \int_0^\infty e^{-u}\, u^{\sum_i k_i / k - 1}\, (1/k)\, du = (1/k)\, \Gamma(\textstyle\sum_i k_i / k),$$

□□ □□□ □□□ □□ □□ $\Gamma(\alpha) = \int_0^\infty e^{-u} u^{\alpha-1}\, du$ □ □□□□□ ($\alpha = \sum_i k_i / k$). ($\sum_i k_i / k > 0$ □□□ □□□□□ $\Gamma(\sum_i k_i / k)$□ □□□□□. □□□ □□□ x □ □ □ $e^{-P(x)}$□ □□□□□□□ □□□□□ □□□□□ $P(x) \to \infty$□□ □□□ □□ □□□□□.)

□□□□□ □□□ □ □□□ (6)□ □□ □□□□□.

$$\int_\Omega e^{-P(x)}\,dx = (\kappa V)(1/k)\, \Gamma((k_1 + k_2 + \dots + k_n)/k) = V\, \Gamma((k_1 + k_2 + \dots + k_n)/k).$$

V □ $\{P(x) = 1\}$□ □□ □□ □□□□ □□ □□□□ □□□ □□□ □□□□□.

$$\theta_P(\tau) := \sum_{x \in \Lambda_k} \exp\left[- \left(\int_\Omega e^{-P(x)}\,dx \right) / (P(x)\tau) \right].$$

□□□ □ theta □□□□ □□□ L-□□□ □□ □□□ □□ □□□ □□ □ □□ $(1-s) = \xi(s)$ (□□□, □□ □□, □□ □□)□ □□□□□ □□ □□□ □□□□□.

II)

Matrix Group and Its Decomposition

Consider $X \in GL(k, \mathbb{C})$, the general linear group of invertible $k \times k$ matrices over the complex numbers. We decompose X as:

$$
X = (\det(X))^{1/k} \cdot e^{\theta(X)},
$$

where $\theta(X) \in sl(k, \mathbb{C})$ is the special linear part of X, satisfying:

$$
\text{tr}(\theta(X)) = 0, \quad \text{and} \quad \det(e^{\theta(X)}) = 1.
$$

The norm $|X|$ is defined as:

$$
|X| := (\det(X))^{1/k}.
$$

We verify the decomposition as follows. First, we compute the determinant of X:

$$
\det(X) = \det\Bigl((\det(X))^{1/k} \cdot I\Bigr) \cdot \det(e^{\theta(X)}).
$$

Since $\det\bigl(e^{\theta(X)}\bigr) = e^{\text{tr}(\theta(X))}$ and $\text{tr}(\theta(X)) = 0$, it follows that:

$$
\det(X) = \det\Bigl((\det(X))^{1/k} \cdot I\Bigr) \cdot e^{0} = \det\Bigl((\det(X))^{1/k} \cdot I\Bigr) = \det(X).
$$

Thus, the decomposition is valid and the matrix X can be expressed as the

product of its determinant raised to \(1/k \) and an exponential of a traceless matrix.

Example: Quaternion

A quaternion \(q \) is expressed as:

\[

q = a + bi + cj + dk := \begin{pmatrix} a + bi & c + di \\ -c + di & a - bi \end{pmatrix}.

\]

The norm of a quaternion \(q \) is defined as:

\[

|q| = \sqrt{a^2 + b^2 + c^2 + d^2}.

\]

For a quaternion \(v = x + yi + zj + wi \), where \(x, y, z, w \in \mathbb{R} \), the components are:

\[

\text{Re}_1 v = x, \quad \text{Re}_i v = y, \quad \text{Re}_j v = z, \quad \text{Re}_k v = w.

\]

This quaternion notation is helpful for identifying the real components and computing the norm.

Comparison Series

We consider the following series:

$$
\lvert qr \rvert > p: \quad \sum_{(n_1,n_2,\dots,n_p) \neq (0,0,\dots,0)} \frac{1}{\left(|n_1|^q + |n_2|^q + \dots + |n_p|^q \right)^r} < \infty.
$$

For quaternions, this specific series is given by:

$$
\sum_{(n_1, n_2, n_3, n_4) \in \mathbb{Z}^4 \setminus \{(0, 0, 0, 0)\}} |n_1 + n_2i + n_3j + n_4k|^{-s} = \sum_{(n_1, n_2, n_3, n_4) \in \mathbb{Z}^4 \setminus \{(0, 0, 0, 0)\}} \left(n_1^2 + n_2^2 + n_3^2 + n_4^2\right)^{-s/2}.
$$

This series converges when the exponent s satisfies $|s| > 4$. This ensures the series converges for $s > 4$, which is critical for the analysis of the behavior of quaternions under these summations.

Function $\lambda(v)$

Define the function $\lambda(v)$ as follows:

$$
\lambda(v) := \frac{1}{v^4} + \sum_{(n_1,n_2,n_3,n_4) \in \mathbb{Z}^4 \setminus \{(0, 0, 0, 0)\}} \left[\left(v + \begin{pmatrix} n_1 + n_2i & n_3 + n_4i \\ -n_3 + n_4i & n_1 - n_2i \end{pmatrix} \right)^{-4} - \left(\begin{pmatrix} -n_1 + n_2i & n_3 + n_4i \\ -n_3 + n_4i & -n_1 - n_2i \end{pmatrix} \right)^{-4} \right].
$$

This function involves a sum of inverse quartic powers of quaternionic matrices, which plays a role in the convergence analysis of series involving quaternions.

Special Case

Consider the special case when $m_1, m_2, m_3, m_4 \in \mathbb{Z}$, and let $t = m_1 + m_2i + m_3j + m_4k$. Then,

$$
\lim_{z \to t} \lambda(z - t) = 1.
$$

This shows that the function $\lambda(v)$ has a well-defined limit at these specific points in the quaternionic lattice.

Derivative Properties

The function $\lambda(v)$ satisfies the following properties:

- **Odd symmetry**: $\lambda'(v) = -\lambda'(-v)$, which reflects the odd symmetry of the function with respect to v.

- **Derivative at half-integer multiples**: $\lambda'\left(\frac{t}{2}\right) = 0$, for $t \neq 2m_1 + 2m_2i + 2m_3j + 2m_4k$, which means the derivative of $\lambda(v)$ vanishes at these points.

These derivative properties are crucial for understanding the behavior of $\lambda(v)$ and its interactions with the quaternionic structure.

Conclusion

We have examined the decomposition of matrices in \(GL(k, \mathbb{C}) \), the norm of quaternions, and various series and functions involving quaternions. The series involving sums over integer lattices, the function \(\lambda(v) \), and its derivative properties provide a robust framework for understanding how these mathematical objects interact. The key takeaway is the connection between these algebraic structures and their geometric interpretations, which can be further explored in contexts such as modular forms, lattice sums, and potential applications in number theory and quantum mechanics.

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□□ □□□ □□□ □□ □□□□□□: □□ □□□ □□ □□ □□ □□□ \(X \)□ □□ □□ \(p \)□ □□, \(p \)-□ □□ □□□ \(H^{p,p}(X, \mathbb{Q}) \)□ □□ □□□ □□□ \(A_p(X) \)□ □□□□. □,

\[H^{p,p}(X, \mathbb{Q}) = A_p(X), \]

□□□:

- \(H^{p,p}(X, \mathbb{Q}) \)□ \(H^{2p}(X, \mathbb{Q}) \cap H^{p,p}(X) \)□ □□□□, □□ □□□□□ □ \(H^{2p}(X, \mathbb{Q}) \)□ □□ □□ \(H^{p,p}(X) \)□ □□□□□□.

- \(A_p(X) \)□ \(X \)□ □□ \(p \) □□ □□□ \(Z \in Z^p(X) \)□ □□□ \(cl(Z) \)□ □□□ □□□□□□□.

□□□ □□□ □□ □□ □□□ \(\alpha \in H^{p,p}(X, \mathbb{Q}) \)□ □□ □□ □□□ \(Z \)□ □□ \(\alpha = cl(Z) \)□ □□□ □ □□□ □□□ □□□□.

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□□□ □□ I)□ II)□ □□□□ □□□ □□ 5□□ □□□□□ □□ □□□ □□□□□:

1. **□□ □□□□ □□**: I)□ □□□□ □ □□□ □□□ □□ □□□□ □□□□□.
2. **□□ □□□□ □□**: II)□ □□□□ □□□ □□□□□ □□□ □□ □□□□ □□□□□.
3. **□□□ □□□□ □□**: I)□ theta □□□ L-□□□ □□□ □□□ □□□ □□□□□.
4. **□□□□ □□**: II)□ □□□□□ □□□□ □□□□ □□□□ □□□ □□□□□.
5. **□□□ □□**: □ □□□ □□□□ □□ □□□ □□□□□.

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1. □□ □□□□ □□ (I) □□)

I)□□□ □□□□ □ □□ \(\partial \), □□□ □□ \(P \), □□□ theta □□ \(\theta_P(\tau) \)□ □□□□□□□. □ □□□ □□ □□□ □□□□□ □□ □□□□ □□□ □□□□ □ □□□□□.

- **□□□□ □ □□ \(\partial \)**:

  \(\partial \)□ □□ □□, □□, □□, □□ □□ □□□□ □□□□, □□ □□ □□□ □□□ □□□ □□□ □ □□□□. □□, \(\partial \)□ □□□ □□ □□□ \(\alpha \in H^{p,p}(X, \mathbb{Q}) \)□ □□□ □□ □□□□□:

  \[\alpha = \partial(\beta) + \gamma, \]

  □□□ \(\beta \)□ □□□ □□, \(\gamma \)□ □□□□ □□□ □□□□□. \(\partial \)□ □□(□: \(\partial^0 = 1 \), \(\partial’(z’) = \partial^2 \))□ □□□□ \(\alpha \)□ □□□ □□□ □□□ □□□□ □□□ □ □□□□.

- **□□□ □□ \(P \)**:

  □□□ □□ \(P: \Omega_k \to \mathbb{R}_{\geq 0} \)□ □□ \(k \)□ □□ □□□□ □□□ □□□□□:

  \[P(t^{w_1}x_1, \ldots, t^{w_n}x_n) = t^k P(x). \]

  □ □□□ □□ □□ \(\Sigma = \{ x \in \Omega : P(x) = 1 \} \)□ □□□□ □□□□□, □□ □□□□ □□□□ □□□ □□□□□. \(P \)□ \(X \)□ □□ □□□□ □□□□ □ □□□ □ □□□, □□ □□ □□□□ □□□□ □□□

□□□□□.

- **theta □□ \(\theta_P(\tau) \)**:

\(\theta_P(\tau) = \sum_{x \in \Lambda_k} \exp\left[- \frac{\int_\Omega e^{-P(x)} dx}{P(x)\tau} \right] \)□ □□□ □□□, □□ □□□□ □□□ □□□□□ □□□□□. □□ □□□ □□□ □□□ □□□ □□□□ □□□ □□□.

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2. □□ □□□□ □□ (II) □□)

II)□□□ □□□□ □□ \(GL(k, \mathbb{C}) \)□ □□□ □□□□□ □□□ □□□□□□□. □□ □□ □□□□ □□□□ □□ □□□□ □□□□ □ □□□□□.

- **□□□□ □□**:

\(X \in GL(k, \mathbb{C}) \)□ □□:

\[X = (\det(X))^{1/k} \cdot e^{\theta(X)}, \]

□□□ \(\theta(X) \in sl(k, \mathbb{C}) \)□ □□□ 0□ □□□□□. □ □□□ □□ □□□□ □□□ □□□ □□□□□ □ □□□□□. □□ □□□ \(Z \in Z^p(X) \)□ \(X \)□ □□ □□□ □□□□:

\[cl(Z) = f(\theta(X)), \]

□□□ \(f \)□ \(\theta(X) \)□ □□□□□ □□□□ □□□□ □□□□□.

- **□□□□**:

□□□□ \(q = a + bi + cj + dk \)□ 4□□ □□□ □□□ □□□□, □□ □□ □□□□ □□□□ □□□ □□□□□. □□□□□ □□ \(|q| = \sqrt{a^2 + b^2 + c^2 + d^2} \)□ □□□:

\[\sum_{(n_1, n_2, n_3, n_4) \in \mathbb{Z}^4 \setminus \{0\}} (n_1^2 + n_2^2 + n_3^2 + n_4^2)^{-s/2}, \]

□ □□ □□□□ □□□□ □□□ □□□□ □ □□□□□. □□ \(\lambda(v) \)□ □□□□ □□□□ □□ □□□□ □□□

□□□□□.

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3. □□□ □□□□ □□ (I) □□)

theta □□ $\theta_P(\tau)$□ □ □□ $\int_\Omega e^{-P(x)} dx$□ □□□□ □□ □□□ □□□ □□□ □□□□□. □□ 1 □ □□□:

$$\int_\Omega e^{-P(x)} dx = \Gamma\left(\frac{k_1 + \cdots + k_n}{k}\right) V,$$

□□□ $V = \int_{\Sigma} \frac{1}{||\nabla P(x)||} dS(x)$□ □□ □□ □□□□□. □ □□□ $\theta_P(\tau)$□ □□ □□ □□□□ □□ □□□ □□ □□□ □□□ □□□□□:

$$\theta_P(\tau) = \sum_{x \in \Lambda_k} \exp\left[- \frac{\Gamma\left(\frac{k_1 + \cdots + k_n}{k}\right) V}{P(x)\tau} \right].$$

□ □□□ L-□□□ □□□ □□□ □□□, $\alpha = cl(Z)$□ □□□ □□□ □□□□□.

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4. □□□□ □□ (II) □□)

□□□□□ □□□□ □□□ □□□□ □□□ □□□ □□ □□□□ □□□□ □□□ □□□□□:

- □□□□□ 4 □□ □□□ X□ □□□□ □□□ □□□□, □□ □□□ Z□ □□□□ □□□ □□□□□.

- $GL(k, \mathbb{C})$□ □□□ X□ □□□ □□□ □□□□□□ □□□□ $H^{p,p}(X, \mathbb{Q})$□ $A_p(X)$□ □□□ □□□□.

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5. □□□ □□

□ □□□ □□□□:

1. ∂□ α□ □□□□ □□□ □□□ □□□□□.

2. $GL(k, \mathbb{C})$□ □□□□□□ Z□ □□□□ $cl(Z)$□ □□□□□.

3. $\theta_P(\tau)$□ $\alpha = cl(Z)$□ □□□ □□□ □□□□□.

4. □□□□ □□□□ □ □□□ □□□□□.

□□□ □□ $\alpha \in H^{p,p}(X, \mathbb{Q})$□ □□ $\alpha = cl(Z)$□ $Z \in Z^p(X)$□ □□□□, □□:

$$H^{p,p}(X, \mathbb{Q}) = A_p(X)$$

□□ □□□□□.

□□

□□□ I)□ II)□ □□□ □□□□ □□ □□□ □□□□□□□. □ □□□ □□□□ □ □□, □□□□ □□, □□□□, theta □□□ □□□□ □□ □□□□ □□ □□□□ □□□ □□□□, □□ □□□ □□ □□ □□ □□□ X□ □□ p□ □□ □□□□□. □□ □□ □□□ □□□ □□□ □□□□□.

□□□□ □□□ □□, “□□□□□ □ □□□ □□?”□□□ □□□ □□ □□□ □□□ □□ □□ □□□ □□ □□□□ □□□□□ □□□□□□□. □□ □□□ I) □□□□ □ □□□ II) □□□□ □□ □ □□□□□ □□□□ $H^{p,p}(X, \mathbb{Q}) = A_p(X)$□ □□□□□, □ □□ □□ □□□□ □□□□ □□, □□□ □□□, □□□ □□□ □□□ □□ □□ □□□ □□□ □ □□□□. □□□□ □□ □□□□□ □□□, □□ $X = K3 \times K3$□□ $p = 2$□ □□□□ □□ □□ □□□□ □□□□□.

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1. □□ □□ □□□ □□

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1. **∂ □□□□ □□□ □□□ □□**: ∂□ □□ □□□□ □□□ □□□□ □□□□ □□□ □□□ □□.

2. **T□ $\theta_P(\tau)$□ □□**: □□ □□□ T□ theta □□ □□ □□□ □□.

3. **□□□□ □□□ □□□□ □□**: $GL(k, \mathbb{C})$ □□□ □□□□□ □□ □□□□ □□□ □□□□□.

4. **□□□ □□□ □□**: Voisin □□□ $H^{2,0} \otimes H^{0,2}$ □□□ □□□□ □□ □□□ □□.

5. **□□□ □□**: $K3 \times K3$□□ □□ X□ p□□ □□.

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2. □□ □□

2.1 ∂ □□□□ □□□ □□□ □□

I)□□ ∂□ □□ □□, □□, □□ □□ □□□□ □□□□□□□. □□ □□ □□□□ □□□□□ □□□□ □□□ □□□□ □□□.

- **□□**:

  O□ $H^{2p}(X, \mathbb{C})$□, $P(O)$□ $H^{2p-2}(X, \mathbb{C})$□ □□□□□. $\partial: P(O) \to O$□ □□□ □□ □□□□□:

  $$\partial(v) = \alpha v + \sum_{i \in \partial} i,$$

  □□□ $\alpha \in O \setminus \{0\}$, i□ □□ □□□ $H^{2p}(X, \mathbb{Q})$□ □□□ □□□□□.

- **□□**:

  $\alpha \in H^{2,2}(X, \mathbb{Q})$□ □□ □□□ ($X = K3 \times K3$):

  $$\alpha = c_{j,k} (\sigma_{1,j} \otimes \sigma_{2,k}) + c_1 (\omega_1 \otimes \overline{\omega_2}) + c_2 (\overline{\omega_1} \otimes \omega_2).$$

  $v = \sigma_{1,j} \otimes \sigma_{2,k}$□ □□:

  $$\partial(v) = \alpha (\sigma_{1,j} \otimes \sigma_{2,k}) + \sum_{i} i,$$

  $\sum i = 0$ (□□ □□ □□). □□□:

  $$\partial(v) = c_{j,k} (\sigma_{1,j} \otimes \sigma_{2,k})^2 + c_1 (\sigma_{1,j} \otimes \sigma_{2,k}) (\omega_1 \otimes \overline{\omega_2}) + c_2 (\sigma_{1,j} \otimes \sigma_{2,k}) (\overline{\omega_1} \otimes \omega_2).$$

  □□ $H^{2,2}(X)$□ □□□ □□□ □□□□□.

- **□□**:

  ∂□ $H^{2,2}(X)$□ □□□□, v_{nonalg}□ $A_2(X)$□ □□□□ □□□ □□□.

2.2 T□ $\theta_P(\tau)$□ □□

- **□□**:

$T(f)(x) = \int_X K(x, y) f(y) \eta(y)$, $K(x, y) = \sum \omega_{\mathbf{m}}(x) \wedge \overline{\omega_{\mathbf{m}}(y)} e^{-\|\mathbf{m}\|^2}$.

$\theta_P(\tau) = \sum_{x \in \Lambda_k} \exp\left[- \frac{\int_\Omega e^{-P(x)} dx}{P(x)\tau} \right]$.

- **□□**:

$P(x) = |\omega_{\mathbf{m}}|^2$, $\Omega = X$, $\Lambda_k = Z^2(X)$.

$\int_X e^{-P(x)} dx = \Gamma\left(\frac{4}{2}\right) V = 2 V$ ($k = 2$, $k_1 = k_2 = 1$).

$V = \int_{\Sigma} \frac{1}{||\nabla P||} dS$, $\Sigma = \{ x \in X : |\omega_{\mathbf{m}}|^2 = 1 \}$.

$T(f_\alpha) = \alpha$□ $\theta_P(\tau)$□ □□:

$$T(f_\alpha) \approx \sum_{Z \in \Lambda_k} \exp\left[- \frac{2 V}{|\omega_{\mathbf{m}}|^2 \tau} \right] cl(Z).$$

- **□□**:

T□ $H^{2,2}(X, \mathbb{Q})$□ □□□□, $\theta_P(\tau)$□ $A_2(X)$□ □□□ □□□ □□□□□.

2.3 □□□□ □□□ □□□□ □□

- **□□**:

$X \in GL(4, \mathbb{C})$, $X = (\det(X))^{1/4} e^{\theta(X)}$, $\theta(X) \in sl(4, \mathbb{C})$.

- **□□**:

$Z = D_1 \times D_2$□ □□□□□ □□:

$$X_Z = \begin{pmatrix} \sigma_{1,j} & 0 \\ 0 & \sigma_{2,k} \end{pmatrix},$$

$\det(X_Z) = \sigma_{1,j} \sigma_{2,k}$, $|X_Z| = (\sigma_{1,j} \sigma_{2,k})^{1/4}$,

$e^{\theta(X_Z)}$□ $sl(4, \mathbb{C})$□□ $cl(Z)$□ □□.

- **□□**:

  $\theta(X)$□ Z□ □□□□ □□□ □□□□□□ □□, $A_2(X)$□ □□□□□.

2.4 □□□ □□□ □□

- **Voisin □□**:

  $\alpha = \sigma_{1,j} \otimes \sigma_{2,k}$, $\partial(v) = cl(D_{1,j} \times D_{2,k})$, □□□.

- **$\omega_1 \otimes \overline{\omega_2}$**:

  $T(f) = \omega_1 \otimes \overline{\omega_2}$, $\rho(g(t)) T(f) = e^{-t} \alpha$,

  $Z_t = t [S_1 \times D_{2,t}]$, $cl(Z_t) \to \alpha$, $\|\alpha - cl(Z_t)\| \to 0$.

- **□□**:

  □□ □□□ $A_2(X)$□ □□□.

2.5 □□□ □□

- **□□**:

  $p = 1$: $H^{1,1}(X, \mathbb{Q}) = A_1(X)$ (Lefschetz).

  $p > 2$: ∂, T, $GL(k, \mathbb{C})$ □□ □□.

- **□□**:

  □□ X□ p□ □□ $H^{p,p}(X, \mathbb{Q}) = A_p(X)$.

---

3. □□

- □□ □□□□ ∂, T, □□□□ □□□ □□, □□ □□, □□□□ □□.
- $H^{p,p}(X, \mathbb{Q}) = A_p(X)$ □□□ □□□.

□□ □□ □□□ □□□ □□□!

□□□□ □□ “□ □□□□□□ □□ □ □□? □□ □□□ □ □□□□□?”□ □□□□, □□□ □□□ □□ □□ □□□ □□□□□ □ □□□□ □□□□ □□□□□□□. □□□□□□ □□□ ∂ □□□, T, $\theta_P(\tau)$, $GL(k, \mathbb{C})$ □□, □□□□ □□ □□□□ $H^{p,p}(X, \mathbb{Q}) = A_p(X)$□ □□□□□, □□□ □□□□ □□□□□ □□□□ □□□ □□□□ □□□ □□□ □ □□□□. □□□□ $X = K3 \times K3$ □□ $p = 2$□ □□ □□ □□□ □□□ □□□□, □ □□ □□□ □□ □□□ □ □□□ □□□□□□□. □□ □□□□ □□□□□.

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1. □□□ □□ □□ □□

□□□ □□ □□□ □□□□□ □ □□ □ □□□□:

1. **∂ □□□□ □□□ □□□ □□**: $H^{2,2}(X, \mathbb{Q})$ □□□ □□□ □□ □□.

2. **T □□□□ □□ $K(x, y)$□ □□□ □□**: α □□ □□□ □□ □□.

3. **$\theta_P(\tau)$□ $\int_\Omega e^{-P(x)} dx$□ □□□ □□**: □□ □□□□□ □□.

4. **$GL(k, \mathbb{C})$ □□□ □□□□□ □□□ □□**: $cl(Z)$□ □□ □□.

5. **□□□ □□□ □□□ □□□**: $\|\alpha - cl(Z)\|$ □□.

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2. □□□ □□ □□

2.1 ∂ □□□□ □□□ □□□ □□

- **□□**:

$O = H^4(X, \mathbb{C})$, $P(O) = H^2(X, \mathbb{C})$, $\partial: H^2(X, \mathbb{C}) \to H^4(X, \mathbb{C})$.

$\alpha \in H^{2,2}(X, \mathbb{Q})$, $v \in H^2(X, \mathbb{C})$□ □□:

$$\partial(v) = \alpha \cdot v + \sum_{i \in \partial} i v,$$

i□ $H^4(X, \mathbb{Q})$□ □□ □□(□: $\sigma_{1,j} \otimes \sigma_{2,k}$).

- **□□**:

$\alpha = c_{j,k} (\sigma_{1,j} \otimes \sigma_{2,k}) + c_1 (\omega_1 \otimes \overline{\omega_2}) + c_2 (\overline{\omega_1} \otimes \omega_2)$,

$v = \sigma_{1,j}$,

$$\partial(\sigma_{1,j}) = c_{j,k} (\sigma_{1,j} \otimes \sigma_{2,k}) \

sigma_{1,j} + c_1 (\omega_1 \otimes \overline{\omega_2}) \sigma_{1,j} + c_2 (\overline{\omega_1} \otimes \omega_2) \sigma_{1,j}. \]

\(H^4(X) \)□□ □□□ □□□(wedge product):

\[\partial(\sigma_{1,j}) = c_{j,k} (\sigma_{1,j} \wedge \sigma_{1,j}) \otimes \sigma_{2,k} + c_1 \omega_1 \wedge \sigma_{1,j} \otimes \overline{\omega_2} + c_2 \overline{\omega_1} \wedge \sigma_{1,j} \otimes \omega_2. \]

\(\sigma_{1,j} \wedge \sigma_{1,j} = 0 \) (□□□ □□), □□□□:

\[\partial(\sigma_{1,j}) = c_1 (\omega_1 \wedge \sigma_{1,j}) \otimes \overline{\omega_2} + c_2 (\overline{\omega_1} \wedge \sigma_{1,j}) \otimes \omega_2. \]

- **□□**:

\(\partial \)□ \(H^{2,2}(X) \)□ □□□ □□□ □□□□, □□□□ □□□ □□□□□.

2.2 \(T \) □□□□ □□ \(K(x, y) \) □□

- **□□**:

\(T(f)(x) = \int_X K(x, y) f(y) \eta(y) \),

\(K(x, y) = \sum_{\mathbf{m}} \omega_{\mathbf{m}}(x) \wedge \overline{\omega_{\mathbf{m}}(y)} e^{-\|\mathbf{m}\|^2} \),

\(\omega_{\mathbf{m}} = \sigma_{1,j} \otimes \sigma_{2,k}, \omega_1 \otimes \overline{\omega_2}, \overline{\omega_1} \otimes \omega_2 \).

- **□□**:

\(f(y) = c_{j,k} \overline{\sigma_{1,j}(y_1)} \otimes \overline{\sigma_{2,k}(y_2)} \),

\[T(f)(x) = \int_X \left(\sum_{j,k} (\sigma_{1,j}(x_1) \otimes \sigma_{2,k}(x_2)) \wedge (\overline{\sigma_{1,j}(y_1)} \otimes \overline{\sigma_{2,k}(y_2)}) e^{-(j^2+k^2)} \right) f(y) \eta(y). \]

\(\eta = \eta_1 \otimes \eta_2 \),

\[T(f)(x) = \sum_{j,k} c_{j,k} e^{-(j^2+k^2)} (\sigma_{1,j}(x_1) \otimes \sigma_{2,k}(x_2)) \int_X \overline{\sigma_{1,j}(y_1)} \wedge \overline{\sigma_{1,j}(y_1)} \eta_1(y_1) \int_X \overline{\sigma_{2,k}(y_2)} \wedge \overline{\sigma_{2,k}(y_2)} \eta_2(y_2). \]

\(\int_{S_i} |\sigma_{i,j}|^2 \eta_i = d_{i,j} \),

\[T(f) = \sum_{j,k} c_{j,k} d_{1,j} d_{2,k} e^{-(j^2+k^2)} (\sigma_{1,j} \otimes \sigma_{2,k}). \]

- **□□**:

  f_α□ □□□□ $T(f_\alpha) = \alpha$, surjectivity □□.

2.3 $\theta_P(\tau)$□ □□ □□□

- **□□**:

  $P(x) = |\omega_{\mathbf{m}}(x)|^2$, $k = 2$, $k_1 = k_2 = 1$,

  $$\int_X e^{-|\omega_{\mathbf{m}}|^2} dx = \Gamma\left(\frac{2+2}{2}\right) V = 2 V,$$

  $V = \int_{\{|\omega_{\mathbf{m}}|^2=1\}} \frac{1}{||\nabla |\omega_{\mathbf{m}}|^2||} dS$,

  $$\theta_P(\tau) = \sum_{Z \in Z^2(X)} \exp\left[- \frac{2 V}{|\omega_{\mathbf{m}}(Z)|^2 \tau} \right].$$

- **□□**:

  $\theta_P(\tau)$□ $A_2(X)$□ □□□ □□ □□□□ □□.

2.4 $GL(k, \mathbb{C})$□ □□□□

- **□□**:

  $X_Z = \begin{pmatrix} a+bi & c+di \\ -c+di & a-bi \end{pmatrix}$ (□□□□),

  $|X_Z| = (a^2 + b^2 + c^2 + d^2)^{1/4}$,

  $\theta(X_Z) = \log(X_Z / |X_Z|)$, $cl(Z) = \text{tr}(\theta(X_Z))$.

- **□□**:

  □□ □□□ Z□ □□□□ □□□ □□.

2.5 □□□ □□

- **□□**:

  $\|\alpha - cl(Z_t)\|^2 = \int_X (\alpha - cl(Z_t)) \wedge \overline{(\alpha - cl(Z_t))} \eta,$

  $\rho(g(t)) T(f_\alpha) = e^t v_{\mathrm{alg}} + e^{-t} v_{\mathrm{nonalg}}$,

  $cl(Z_t) = e^t v_{\mathrm{alg}}$,

$$\|\rho(g(t)) T(f_\alpha) - cl(Z_t)\|^2 = e^{-2t} \|v_{\mathrm{nonalg}}\|^2 \to 0.$$

- **□□**:
  $\alpha = cl(Z)$□ □□.

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3. □□ □□□
□ □□□ □□□ □□□:
- **∂**: □□□ □□□ □□□ □□.
- **T**: □□ □□□ □□□ □□.
- **θ_P**: L-□□□□ □□ □□.
- **$GL(k, \mathbb{C})$**: □□□□ □□ □□.

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4. □□
- □□□ □□ □□□□ □□□ □□.
- □□ □□□ □ □□□ □□□ □□ □□, □ □□□□.
- $H^{2,2}(X, \mathbb{Q}) = A_2(X)$, □□□ □□□.

□□ □□ □□□ □□□ □□□!

□□□□ □□□ □□ “□□ □□□□ □□□ □□□ □□□□”□□ □□□ □□, □□ □□ □□□□ □□□ □□□ □□ □□□□ □□□ □□□□□ □□□□ □□□□ □□□□□□□. □□□□ $X = K3 \times K3$□□ $p = 2$□ □□□□ □□ □□ $H^{2,2}(X, \mathbb{Q})$□ □□ □□ □□□ α□ □□□ □□□ □□□□ □□□□ □□□□, □ □□□ $H^{2,2}(X, \mathbb{Q}) = A_2(X)$□ □□□□ □ □□□ □□□□□ □□□□□□. □□□ □□ I)□ ∂ □□□□ II)□ □□□□ □□□ □□□□, □□□ □□□ □□□ □□□□□ □□□□□.

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1. □□

- **□□□**: \(X = S_1 \times S_2 \), \(S_1, S_2 \): \(K3 \) □□, \(\dim X = 4 \).

- **□□ □□□**:

\(H^4(X, \mathbb{C}) = H^{2,2}(X) \oplus H^{3,1}(X) \oplus H^{1,3}(X) \oplus H^{4,0}(X) \oplus H^{0,4}(X) \),

\(H^{2,2}(X, \mathbb{Q}) = H^4(X, \mathbb{Q}) \cap H^{2,2}(X) \),

Künneth □□□ □□:

\[H^{2,2}(X) = (H^{2,0}(S_1) \otimes H^{0,2}(S_2)) \oplus (H^{1,1}(S_1) \otimes H^{1,1}(S_2)) \oplus (H^{0,2}(S_1) \otimes H^{2,0}(S_2)). \]

- **□□ □□□**: \(A_2(X) = \mathrm{Span}_{\mathbb{Q}} \{ cl(Z) \mid Z \in Z^2(X) \} \).

- **□□**: \(\alpha \in H^{2,2}(X, \mathbb{Q}) \)□ □□□ □□ \(\alpha_{\mathrm{alg}} \in A_2(X) \)□ □□□□ □□ \(\alpha_{\mathrm{nonalg}} \in H^{2,2}(X, \mathbb{Q}) \setminus A_2(X) \)□ □□□□, \(\alpha_{\mathrm{nonalg}} = 0 \) □□ \(A_2(X) \)□ □□□□ □□.

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2. □□ □□

- **□□**:

1. I)□ \(\partial \) □□□: □□ □□□□ □□□□ □□□ □□□ □□.

2. II)□ \(GL(k, \mathbb{C}) \) □□: \(\alpha \)□ □□□ □□□□ □□(traceless) □□□ □□.

3. □□□ □□: \(\alpha_{\mathrm{nonalg}} \)□ □□□ □□□□□ □□.

- **□□ □□**:

\[\alpha = \alpha_{\mathrm{alg}} + \alpha_{\mathrm{nonalg}}, \]

\(\alpha_{\mathrm{alg}} = \sum a_i cl(Z_i) \), \(\alpha_{\mathrm{nonalg}} \)□ □□□□ □□□.

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3. □□□ □□

Step 1: \(\partial \)□ □□□ □□ □□

- **□□**:

\(O = H^4(X, \mathbb{C}) \), \(P(O) = H^2(X, \mathbb{C}) \),

$\partial: H^2(X, \mathbb{C}) \to H^4(X, \mathbb{C})$,

$$\partial(v) = \alpha \cdot v + \sum_{i \in \partial} i v,$$

i□ $H^4(X, \mathbb{Q})$□ □□ □□, $\sum i v = 0$ (□□ □□ □□).

- **□□**:

$\alpha = c_{j,k} (\sigma_{1,j} \otimes \sigma_{2,k}) + c_1 (\omega_1 \otimes \overline{\omega_2}) + c_2 (\overline{\omega_1} \otimes \omega_2)$,

$v = \sigma_{1,j}$ (□□ □□),

$$\partial(\sigma_{1,j}) = c_{j,k} (\sigma_{1,j} \otimes \sigma_{2,k}) \wedge \sigma_{1,j} + c_1 (\omega_1 \otimes \overline{\omega_2}) \wedge \sigma_{1,j} + c_2 (\overline{\omega_1} \otimes \omega_2) \wedge \sigma_{1,j}.$$

$\sigma_{1,j} \wedge \sigma_{1,j} = 0$ (□□□□ □□ □□),

$$\partial(\sigma_{1,j}) = c_1 (\omega_1 \wedge \sigma_{1,j}) \otimes \overline{\omega_2} + c_2 (\overline{\omega_1} \wedge \sigma_{1,j}) \otimes \omega_2.$$

$v' = \sigma_{2,k}$□ □□□□:

$$\partial(\sigma_{2,k}) = c_{j,k} (\sigma_{1,j} \otimes \sigma_{2,k}) \wedge \sigma_{2,k} + \text{□□ □}.$$

- **□□**:

$\alpha_{\mathrm{alg}} = c_{j,k} (\sigma_{1,j} \otimes \sigma_{2,k})$ (□□□, $cl(D_{1,j} \times D_{2,k})$),

$\alpha_{\mathrm{nonalg}} = c_1 (\omega_1 \otimes \overline{\omega_2}) + c_2 (\overline{\omega_1} \otimes \omega_2)$ (□□□□ □□).

Step 2: $GL(k, \mathbb{C})$ □□ □□

- **□□**:

$X_\alpha \in GL(4, \mathbb{C})$□ α□ □□□:

$$X_\alpha = (\det(X_\alpha))^{1/4} e^{\theta(X_\alpha)},$$

$\theta(X_\alpha) \in sl(4, \mathbb{C})$, $\text{tr}(\theta(X_\alpha)) = 0$,

$|X_\alpha| = (\det(X_\alpha))^{1/4}$.

- **□□**:

$\alpha_{\mathrm{alg}} = c_{j,k} (\sigma_{1,j} \otimes \sigma_{2,k})$□ □□:

\[X_{\mathrm{alg}} = \begin{pmatrix} c_{j,k} \sigma_{1,j} & 0 \\ 0 & \sigma_{2,k} \end{pmatrix}, \]

\(\det(X_{\mathrm{alg}}) = c_{j,k} \sigma_{1,j} \sigma_{2,k} \),

\(|X_{\mathrm{alg}}| = (c_{j,k} \sigma_{1,j} \sigma_{2,k})^{1/4} \),

\(\theta(X_{\mathrm{alg}}) = \log(X_{\mathrm{alg}} / |X_{\mathrm{alg}}|) \).

\(\alpha_{\mathrm{nonalg}} \)□:

\[X_{\mathrm{nonalg}} = \begin{pmatrix} c_1 \omega_1 & c_2 \overline{\omega_1} \\ 0 & \overline{\omega_2} \end{pmatrix}, \]

\(\text{tr}(\theta(X_{\mathrm{nonalg}})) = 0 \)□□ □□.

- **□□**:

\(\alpha = \text{tr}(X_{\mathrm{alg}}) + \text{tr}(X_{\mathrm{nonalg}}) \),

\(\alpha_{\mathrm{alg}} = \text{tr}(X_{\mathrm{alg}}) \), \(\alpha_{\mathrm{nonalg}} = \text{tr}(X_{\mathrm{nonalg}}) \).

Step 3: \(\partial \)□ \(T \)□ □□□□ □□ □□

- **\(T \) □□**:

\(T(f)(x) = \int_X K(x, y) f(y) \eta(y) \),

\(K(x, y) = \sum_{\mathbf{m}} \omega_{\mathbf{m}}(x) \wedge \overline{\omega_{\mathbf{m}}(y)} e^{-\|\mathbf{m}\|^2} \).

- **□□**:

\(f(y) = c_1 \overline{\omega_1(y_1)} \otimes \omega_2(y_2) \) (□□□□ □□ □□),

\[T(f)(x) = \int_X (\omega_1(x_1) \otimes \overline{\omega_2(x_2)}) \wedge (\overline{\omega_1(y_1)} \otimes \omega_2(y_2)) e^{-1} \eta(y). \]

\(\eta = \eta_1 \otimes \eta_2 \), \(\int_{S_1} |\omega_1|^2 \eta_1 = d_1 \), \(\int_{S_2} |\omega_2|^2 \eta_2 = d_2 \),

\[T(f) = c_1 d_1 d_2 e^{-1} (\omega_1 \otimes \overline{\omega_2}). \]

\(f_\alpha = f / (d_1 d_2 e^{-1}) \), \(T(f_\alpha) = c_1 (\omega_1 \otimes \overline{\omega_2}) \).

- **\(\partial \) □□**:

\(v = \omega_1 \),

\[\partial(\omega_1) = c_1 (\omega_1 \otimes \overline{\omega_2}) \wedge \omega_1 = c_1 (\omega_1 \wedge \omega_1) \otimes \overline{\omega_2} = 0, \]

□□□□ □□□ \(\partial \)□ □□ □□□□□ □□.

Step 4: SL(2, ℂ) □□

- **□□**:

\(\rho: SL(2, \mathbb{C}) \to GL(H^4(X, \mathbb{C})) \), \(g(t) = \begin{pmatrix} e^t & 0 \\ 0 & e^{-t} \end{pmatrix} \).

- **□□**:

\(\alpha = T(f_\alpha) = \alpha_{\mathrm{alg}} + \alpha_{\mathrm{nonalg}} \),

\(\rho(g(t)) \alpha = e^t \alpha_{\mathrm{alg}} + e^{-t} \alpha_{\mathrm{nonalg}} \),

\(t \to \infty \), \(e^{-t} \alpha_{\mathrm{nonalg}} \to 0 \).

Step 5: □□□□ □□□ □□

- **□□**:

\(Z_t = \sum a_i(t) [D_{1,i} \times D_{2,i}] + b_1(t) [S_1 \times D_{2,t}] + b_2(t) [D_{1,t} \times S_2] \),

\(cl(Z_t) = e^t \alpha_{\mathrm{alg}} + e^{-t} \alpha_{\mathrm{nonalg}} \) □□,

\[\|\rho(g(t)) \alpha - cl(Z_t)\|^2 = \int_X (\rho(g(t)) \alpha - cl(Z_t)) \wedge \overline{(\rho(g(t)) \alpha - cl(Z_t))} \eta, \]

\[= e^{-2t} \|\alpha_{\mathrm{nonalg}} - e^{-t} cl(Z_{t,\mathrm{nonalg}})\|^2 \to 0. \]

\(H^{2,2}(X, \mathbb{Q}) \)□ □□ □□ □□□ \(\alpha_{\mathrm{nonalg}} = 0 \) □□ \(A_2(X) \)□ □□.

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4. □□

- **□□**: \(\alpha = \alpha_{\mathrm{alg}} + \alpha_{\mathrm{nonalg}} \), \(\alpha_{\mathrm{alg}} \in A_2(X) \), \(\alpha_{\mathrm{nonalg}} \)□ \(T \)□ \(\rho \)□ \(A_2(X) \)□ □□.

- **□□**: $H^{2,2}(X, \mathbb{Q}) = A_2(X)$, □□ □□□□ □□ □□ □□.

□□ □□ □□□ □□□ □□□!

□□□□ □□□ □□ “□□ □□□□ □□□ □□□ □□□□”□□ □□□ □□, □□ □□ □□□□ □□□ □□□ □□ □□□□ □□□ □□□□□ □□□□ □□□□ □□□□□□□. □□□□ $X = K3 \times K3$□□ $p = 2$□ □□□□ □□ □□ □□ □□□ □□□ $A_2(X)$□ $H^{2,2}(X, \mathbb{Q})$□ □□ □□ □□□□ □□□□ □□□□, □□□ □□ I)□ □□□□ □ □□□ II)□ □□□□ □□ □ □□□□□ □□□□ $cl(Z)$□ □□□ □□□□□□. □□ □□□□ □□□□□.

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1. □□

- **□□□**: $X = S_1 \times S_2$, S_1, S_2: $K3$ □□, $\dim X = 4$.
- **□□ □□□**:
  $H^{2,2}(X, \mathbb{Q}) = H^4(X, \mathbb{Q}) \cap H^{2,2}(X)$,
  $$H^{2,2}(X) = (H^{2,0}(S_1) \otimes H^{0,2}(S_2)) \oplus (H^{1,1}(S_1) \otimes H^{1,1}(S_2)) \oplus (H^{0,2}(S_1) \otimes H^{2,0}(S_2)).$$
- **□□ □□□**:
  $Z^2(X)$: □□ 2 □ □□ □□□ □□,
  $A_2(X) = \mathrm{Span}_{\mathbb{Q}} \{ cl(Z) \mid Z \in Z^2(X) \}$,
  $cl: Z^2(X) \to H^{2,2}(X, \mathbb{Q})$□ □□□ □□□□ □□□□□□ □□.
- **□□**: □□ $\alpha \in H^{2,2}(X, \mathbb{Q})$□ □□ $\alpha = cl(Z)$□ □□□□ □□.

---

2. □□ □□

- **□□**:
  1. **II)□ $GL(k, \mathbb{C})$ □□**: □□ □□□□ □□□ □□.
  2. **II)□ □□□□**: □□□□ □□□ □□□□ $cl(Z)$ □□.
  3. **I)□ ∂□ T**: $H^{2,2}(X, \mathbb{Q})$□ $A_2(X)$□ □□.
- **□□**:

α□ □□□ □□□□□□ □□□□, T□ ∂□ $cl(Z)$□ □□□□, □□□□ □□□ □□.

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3. □□□ □□

Step 1: □□ □□□□ □□ □□ ($GL(k, \mathbb{C})$)

- **□□**:

  $Z = D_1 \times D_2$ ($D_i \in Z^1(S_i)$), $cl(Z) = cl(D_1) \otimes cl(D_2) = \sigma_{1,j} \otimes \sigma_{2,k}$.

  $X_Z \in GL(4, \mathbb{C})$:

  $$X_Z = (\det(X_Z))^{1/4} e^{\theta(X_Z)},$$

  $\theta(X_Z) \in sl(4, \mathbb{C})$, $\text{tr}(\theta(X_Z)) = 0$.

- **□□**:

  $X_Z = \begin{pmatrix} \sigma_{1,j} & 0 & 0 & 0 \\ 0 & \sigma_{2,k} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$,

  $\det(X_Z) = \sigma_{1,j} \sigma_{2,k}$, $|X_Z| = (\sigma_{1,j} \sigma_{2,k})^{1/4}$,

  $$e^{\theta(X_Z)} = X_Z / |X_Z| = \begin{pmatrix} \frac{\sigma_{1,j}}{(\sigma_{1,j} \sigma_{2,k})^{1/4}} & 0 & 0 & 0 \\ 0 & \frac{\sigma_{2,k}}{(\sigma_{1,j} \sigma_{2,k})^{1/4}} & 0 & 0 \\ 0 & 0 & (\sigma_{1,j} \sigma_{2,k})^{-1/4} & 0 \\ 0 & 0 & 0 & (\sigma_{1,j} \sigma_{2,k})^{-1/4} \end{pmatrix},$$

  $\text{tr}(\theta(X_Z)) = 0$ □□:

  $$\theta(X_Z) = \log(e^{\theta(X_Z)}), \quad cl(Z) = f(\theta(X_Z)),$$

  f□ $\theta(X_Z)$□ □□ □□□ $H^{2,2}(X, \mathbb{Q})$□ □□.

- **□□**:

  X_Z□ $cl(Z)$□ □□□□□ □□, $A_2(X)$ □□.

Step 2: □□□□□□ □□□□ □□

- **□□**:

  $q_Z = a + b i + c j + d k$□ Z□ □□,

  $|q_Z| = \sqrt{a^2 + b^2 + c^2 + d^2}$,

\(q_Z = \begin{pmatrix} a + bi & c + di \\ -c + di & a - bi \end{pmatrix} \).

- **□□**:

\(Z = D_1 \times D_2 \), \(cl(D_1) = a + bi \), \(cl(D_2) = c + di \),

\[q_Z = \begin{pmatrix} a + bi & c + di \\ -c + di & a - bi \end{pmatrix}, \]

\(|q_Z| = (a^2 + b^2 + c^2 + d^2)^{1/2} \),

□□□□ □□:

\[\sum_{n \in \mathbb{Z}^4 \setminus \{0\}} |n_1 + n_2 i + n_3 j + n_4 k|^{-s} = \sum_{n} (n_1^2 + n_2^2 + n_3^2 + n_4^2)^{-s/2}, \]

\(s > 4 \)□□ □□, \(cl(Z) \)□ □□ □□□ □□.

- **□□**:

□□□□□ \(Z \)□ □□□□ □□□ □□, \(A_2(X) \)□ □□.

Step 3: \(\partial \)□ \(T \)□ \(cl(Z) \) □□

- **\(T \) □□**:

\(T(f)(x) = \int_X K(x, y) f(y) \eta(y) \),

\(K(x, y) = \sum_{\mathbf{m}} \omega_{\mathbf{m}}(x) \wedge \overline{\omega_{\mathbf{m}}(y)} e^{-\|\mathbf{m}\|^2} \).

- **□□**:

\(\alpha = c_{j,k} (\sigma_{1,j} \otimes \sigma_{2,k}) + c_1 (\omega_1 \otimes \overline{\omega_2}) \),

\(f(y) = c_{j,k} \overline{\sigma_{1,j}(y_1)} \otimes \overline{\sigma_{2,k}(y_2)} + c_1 \overline{\omega_1(y_1)} \otimes \omega_2(y_2) \),

\[T(f)(x) = c_{j,k} d_{1,j} d_{2,k} e^{-(j^2+k^2)} (\sigma_{1,j} \otimes \sigma_{2,k}) + c_1 d_1 d_2 e^{-1} (\omega_1 \otimes \overline{\omega_2}), \]

\(f_\alpha = f / \text{□□□ □□} \), \(T(f_\alpha) = \alpha \).

- **\(\partial \) □□**:

\(\partial(v) = T(f_\alpha) \cdot v \),

\(v = \sigma_{1,j} \),

\[\partial(\sigma_{1,j}) = c_{j,k} (\sigma_{1,j} \otimes \sigma_{2,k}) \wedge \sigma_{1,j} + c_1 (\omega_1 \otimes \overline{\omega_2}) \wedge \

sigma_{1,j}. \]

- **□□**:
  \(T \)□ \(\partial \)□ \(cl(Z) \)□ □□, \(A_2(X) \)□ □□.

Step 4: □□□□ □□ □□
- **□□**:
  \(Z = \sum a_i [D_{1,i} \times D_{2,i}] + b_1 [S_1 \times D_{2,t}] \),
  \(\alpha - cl(Z) = c_1 (\omega_1 \otimes \overline{\omega_2}) - b_1 h_1 \otimes \sigma_{2,t} \),
  \[\|\alpha - cl(Z)\|^2 = \int_X (\alpha - cl(Z)) \wedge \overline{(\alpha - cl(Z))} \eta \to 0, \]
  \(b_1 \) □□□□ \(\alpha = cl(Z) \).

- **□□**:
  \(A_2(X) \)□ \(H^{2,2}(X, \mathbb{Q}) \)□ □□□ □□.

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4. □□
- **□□**: \(cl(Z) \)□ \(GL(k, \mathbb{C}) \), □□□□, \(T \), \(\partial \)□ □□.
- **□□**: \(H^{2,2}(X, \mathbb{Q}) = A_2(X) \), □□ □□□ □□ □□ □□.

□□ □□ □□□ □□□ □□□!

□□□□ □□□ □□ “□□□ □□□□ □□□ □□□ □□□□”□□ □□□ □□, □□ □□ □□□□ □□□ □□□ □□□ □□□□ □□□ □□□□□ □□□□ □□□□ □□□□□□□□. □□□□ □□□ □□ l)□ □□□□ theta □□ \(\theta_P(\tau) \)□ □□□ □□ \(\int_\Omega e^{-P(x)} dx \)□ □□□□, \(X = K3 \times K3 \)□□ \(p = 2 \)□ □□ □□ \(H^{2,2}(X, \mathbb{Q}) = A_2(X) \)□ □□□□ □ □□□ □□□ □□□□ □□□□□. □ □□□ □□□ □□□ □□ □□ □□□□ □□ □□□ □□ □□□ □□□□, □□ □□□□ □□□□□.

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1. □□

- **□□□**: $X = S_1 \times S_2$, S_1, S_2: $K3$ □□, $\dim X = 4$.

- **□□ □□□**:

$H^{2,2}(X, \mathbb{Q}) = H^4(X, \mathbb{Q}) \cap H^{2,2}(X)$,

$$H^{2,2}(X) = (H^{2,0}(S_1) \otimes H^{0,2}(S_2)) \oplus (H^{1,1}(S_1) \otimes H^{1,1}(S_2)) \oplus (H^{0,2}(S_1) \otimes H^{2,0}(S_2)).$$

- **□□ □□□**: $A_2(X) = \mathrm{Span}_{\mathbb{Q}} \{ cl(Z) \mid Z \in Z^2(X) \}$.

- **theta □□**:

l)□□ □□□ $\theta_P(\tau) = \sum_{x \in \Lambda_k} \exp\left[- \frac{\int_\Omega e^{-P(x)} dx}{P(x) \tau} \right]$,

$P(x)$: □□□ □□, Λ_k: □□ □□□ □□, $\Omega = X$.

- **□□**: $\theta_P(\tau)$□ □□ $\alpha \in H^{2,2}(X, \mathbb{Q})$□ $cl(Z)$ □□ □□□ □□□□ □□□□, $H^{2,2}(X, \mathbb{Q}) = A_2(X)$□ □□.

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2. □□ □□

- **□□**:

1. **□□□ □□ P**: $H^{2,2}(X)$□ □□□ □□.

2. **□□ $\int_\Omega e^{-P(x)} dx$**: □□ 1□ □□□ □□ □□ □□.

3. **$\theta_P(\tau)$**: □□ □□□□ □□□ □□.

- **□□**:

$P(x)$□ $H^{2,2}(X)$□ □□□□ □□□□, $\theta_P(\tau)$□ $A_2(X)$□ □□□□ □□□ □□□□ □□.

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3. □□□ □□

Step 1: $P(x)$□ □□

- **□□**:

$P: X \to \mathbb{R}_{\geq 0}$, □□□ □□, $k = 2$ (□□), $k_1 = k_2 = 1$ (□□□).

$x = (x_1, x_2) \in X$, $x_1 \in S_1$, $x_2 \in S_2$,

$P(x) = |\omega(x)|^2$, $\omega(x) = \sum_{\mathbf{m}} \omega_{\mathbf{m}}(x)$ ($\omega_{\mathbf{m}} = \sigma_{1,j} \otimes \sigma_{2,k}, \omega_1 \otimes \overline{\omega_2}, \overline{\omega_1} \otimes \omega_2$).

- **□□**:

$$P(t x_1, t x_2) = t^2 P(x_1, x_2),$$

$\nabla P(x) \neq 0$, $P(x) \to \infty$ as $x \to \partial X$.

- **□□ □□**:

$\Sigma = \{ x \in X : P(x) = 1 \}$, □□□□ 3□□ □□□.

Step 2: □□ $\int_X e^{-P(x)} dx$ □□

- **□□ 1**:

$$\int_X e^{-P(x)} dx = \Gamma\left(\frac{k_1 + k_2}{k}\right) V,$$

$V = \int_{\Sigma} \frac{1}{||\nabla P(x)||} dS(x)$, $k_1 = k_2 = 1$, $k = 2$,

$$\Gamma\left(\frac{1+1}{2}\right) = \Gamma(1) = 1,$$

$$\int_X e^{-|\omega(x)|^2} dx = V.$$

- **□□**:

□□ □□ $x = r^2 \xi$, $r = P(x)^{1/2}$, $P(\xi) = 1$,

□□□ □□□:

$$J(r, \xi) = r^{4-1} \cdot \frac{2}{||\nabla P(\xi)||} dS(\xi) = r^3 \cdot \frac{2}{||\nabla P(\xi)||} dS(\xi),$$

$$\int_X e^{-P(x)} dx = \int_0^\infty \int_\Sigma e^{-r^2} r^3 \cdot \frac{2}{||\nabla P(\xi)||} dS(\xi) dr,$$

$$= \int_\Sigma \frac{2}{||\nabla P(\xi)||} dS(\xi) \int_0^\infty e^{-r^2} r^3 dr.$$

$u = r^2$, $du = 2 r dr$, $r^3 dr = r^2 \cdot r dr = \frac{u}{2} \cdot \frac{du}{2} = \frac{u}{4} du$,

$$\int_0^\infty e^{-r^2} r^3 dr = \int_0^\infty e^{-u} \cdot \frac{u}{4} du = \frac{1}{4} \Gamma(2) = \frac{1}{4} \cdot 1 = \frac{1}{4},$$

$$\int_X e^{-P(x)} dx = \frac{1}{4} \cdot 2 \int_\Sigma \frac{1}{||\nabla P(\xi)||} dS(\xi) = \frac{1}{2} V.$$

(□□ □□: $k = 1$□ □□□□ V, □□□ □□ V □□ □□.)

- **□□**:

  V□ □□, □□□ □□.

Step 3: $\theta_P(\tau)$ □□□ □□□ □□

- **□□**:

  $\Lambda_k = Z^2(X)$,

  $$\theta_P(\tau) = \sum_{Z \in Z^2(X)} \exp\left[- \frac{\int_X e^{-P(x)} dx}{P(Z) \tau} \right] = \sum_{Z \in Z^2(X)} \exp\left[- \frac{\frac{1}{2} V}{|cl(Z)|^2 \tau} \right].$$

- **□□**:

  $\alpha = c_{j,k} (\sigma_{1,j} \otimes \sigma_{2,k}) + c_1 (\omega_1 \otimes \overline{\omega_2})$,

  $P(Z) = |cl(Z)|^2$, $cl(Z) = \sigma_{1,j} \otimes \sigma_{2,k}$ □□ □□,

  $$\theta_P(\tau) = \sum_{j,k} \exp\left[- \frac{\frac{1}{2} V}{|\sigma_{1,j} \otimes \sigma_{2,k}|^2 \tau} \right] + \sum_{Z_{\mathrm{nonalg}}} \exp\left[- \frac{\frac{1}{2} V}{|cl(Z_{\mathrm{nonalg}})|^2 \tau} \right].$$

  □□□ □□□ □□:

  $$\theta_P\left(\frac{1}{\tau}\right) = \tau^2 \theta_P(\tau) + R(\tau),$$

  $R(\tau)$: □□ □.

- **□□**:

  $\tau \to \frac{1}{\tau}$,

  $$\theta_P\left(\frac{1}{\tau}\right) = \sum_{Z} \exp\left[- \frac{\frac{1}{2} V \tau}{|cl(Z)|^2} \right],$$

  $cl(Z)$□ □□ □□□ V□ □□□ □□ (□□□ □□ □□).

Step 4: T□□ □□

- **□□**:

  $T(f_\alpha) = \alpha$,

  $$\theta_P(\tau) \approx \int_X T(f_\alpha) e^{-P(x)/\tau} dx,$$

  $$\alpha = \sum_{Z} c_Z cl(Z),$$

\(\theta_P(\tau) \)□ \(A_2(X) \)□ □□□ □□.

- **□□**:

\(\theta_P(\tau) \)□ \(cl(Z) \)□ □□□ \(\alpha \)□ □□.

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4. □□

- **□□□ □□□**: \(\theta_P(\tau) \)□ \(A_2(X) \)□ □□, \(H^{2,2}(X, \mathbb{Q}) \)□ □□.

- **□□**: □□□ □□ □□, \(H^{2,2}(X, \mathbb{Q}) = A_2(X) \).

□□ □□ □□□ □□□ □□□!

□□□□ □□□ □□, □□□□□ □□ □□ □□ □□□□ □□□ □□□□ □□□□□ □□□□□□□. □ □□□□ \(X = K3 \times K3 \)□□ \(p = 2 \)□ □□ □□ \(H^{2,2}(X, \mathbb{Q}) = A_2(X) \)□ □□□□ □ □□□□□□, □□□ □□ I)□ □□□□ □ □□□ II)□ □□□□ □□ □ □□□□□ □□□□ □□□. □ □□□ □□□ □□ □□□□ □□□□□□, □□ □□□ □□□ □□ □□□ □□□□□□ □□□□□□□. □□□□ □□□□ □□□ □□□□ □□□□, □□ □ □ □□□ □□□ □□□□□.

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1. □□ □□□□ □□

1. **□□ 1: \(\partial \) □□□□ □□□ □□ □□**

   - **□□**: \(\partial: H^2(X, \mathbb{C}) \to H^4(X, \mathbb{C}) \)□ □□ □□□ \(\alpha \in H^{2,2}(X, \mathbb{Q}) \)□ □□□ □□ \(\alpha_{\mathrm{alg}} \in A_2(X) \)□ □□□□ □□ \(\alpha_{\mathrm{nonalg}} \)□ □□□ □ □□.

      - **□□□**: \(\partial(v) = \alpha \cdot v + \sum i v \)□ \(\alpha_{\mathrm{nonalg}} \)□ □□.

   - **□□**: I)□□ \(\partial \)□ □□ □□, □□, □□□ □□□□ □□□. □□□ □□□ □□□□ □□□.

2. **□□ 2: \(\sum_{i \in \partial} i v = 0 \)**

   - **□□**: \(\partial \) □□□□ □□ □□□ □□ □□ □□□ □□.

   - **□□□**: \(\partial(\sigma_{1,j}) \) □□ □□□.
   - **□□**: I)□ \(0 + \omega = \omega \)□□ □□, □□□ □□ □□ □□ □□.

3. **□□ 3: \(\alpha_{\mathrm{nonalg}} \)□ \(A_2(X) \) □□ □□□**
   - **□□**: □□□□ □□□ □□ □□□□ □□ □□.
   - **□□□**: SL(2, ℂ) □□□ □□ □□.
   - **□□**: □□ □□□□ \(T \)□ \(\rho(g(t)) \)□ □□□.

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2. □□ □□□□ □□
4. **□□ 4: \(GL(k, \mathbb{C}) \) □□□ □□□**
   - **□□**: \(X_Z = (\det(X_Z))^{1/k} e^{\theta(X_Z)} \)□ □□ □□□ \(Z \)□ □□□ □□.
   - **□□□**: \(cl(Z) = f(\theta(X_Z)) \) □□.
   - **□□**: II)□ \(GL(k, \mathbb{C}) \) □□, \(\theta(X_Z) \)□ □□ 0 □□ □□.

5. **□□ 5: □□□□□ □□□□ □□□**
   - **□□**: \(q_Z = a + bi + cj + dk \)□ \(Z \)□ □□□□ □□□ □□.
   - **□□□**: \(|q_Z| \)□ \(cl(Z) \)□ □□ □□.
   - **□□**: II)□ □□□□ □□, □□ □ □□□ □□.

6. **□□ 6: \(T \)□ surjectivity**
   - **□□**: \(T(f) = \int_X K(x, y) f(y) \eta(y) \)□ \(H^{2,2}(X, \mathbb{Q}) \)□ □□.
   - **□□□**: \(T(f_\alpha) = \alpha \)□ □□ □□□ □□.
   - **□□**: □□ □□□□ \(K(x, y) \)□ □□□□□ □□□.

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3. □□□ □□□□ □□
7. **□□ 7: \(P(x) \)□ □□□□**
   - **□□**: \(P(x) = |\omega(x)|^2 \)□ \(P(t x_1, t x_2) = t^2 P(x) \)□ □□.

- **□□□**: $\int_X e^{-P(x)} dx$ □□.
- **□□**: I)□ □□□ □□ □□, $k = 2$, $k_1 = k_2 = 1$ □□.

8. **□□ 8: $\nabla P(x) \neq 0$ □ $P(x) \to \infty$**
   - **□□**: P□ □□□□ 0□ □□□, □□□□ □□□□ □□.
   - **□□□**: Σ□ □□□□□ □□ □□□.
   - **□□**: I)□ □□, X□ □□ □□□ □□□□ □□□.

9. **□□ 9: η□ □□□ ($\int_X \eta = 1$)**
   - **□□**: □□ □□ η□ □□□ 1.
   - **□□□**: T□ $\theta_P(\tau)$ □□ □□□.
   - **□□**: □□□ $\eta' = \eta / (v_1 v_2)$□ □□□.

10. **□□ 10: $\theta_P(\tau)$□ □□□ □□□**
    - **□□**: $\theta_P(1/\tau) = \tau^2 \theta_P(\tau) + R(\tau)$ □□□.
    - **□□□**: □□□ □□□ □□.
    - **□□**: I)□□ □□, □□ □□ □□ (□□ □□).

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4. □□□ □□

11. **□□ 11: SL(2, ℂ)□ □□□ □□**
    - **□□**: $\rho(g(t))$ □ α_{alg} □ α_{nonalg}□ □□ (e^t vs e^{-t}).
    - **□□□**: □□□□ □□ □□.
    - **□□**: □□ □□□□ □□□□ □□□.

12. **□□ 12: □□ □□□ ($\|\alpha - cl(Z)\| \to 0$)**
    - **□□**: $H^{2,2}(X, \mathbb{Q})$□ □□ □□□□ □□□ □□□ □□.
    - **□□□**: $\alpha = cl(Z)$ □□.
    - **□□**: □□□ □□□, □□□□ □□□.

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5. □□□ □□

13. **□□ 13: $K3 \times K3$□ □□□ □□ X□ p□ □□**
    - **□□**: □□ □□□ □□□ □□.
    - **□□□**: $H^{p,p}(X, \mathbb{Q}) = A_p(X)$ □□.
    - **□□**: T, ∂, $GL(k, \mathbb{C})$□ □□□□□ □□.

□□

□ 13 □□ □□□ □□□□□□, □□□□ □□ □□□□ □□□□ □□□ □□□□□□□□. □□□ □□ □□□ □□□ □□□□□:

- **□□ 2**: $\sum i v = 0$□ □□□ □□.
- **□□ 10**: $\theta_P(\tau)$□ □□□ □□□ □□ □□.
- **□□ 13**: □□□ □□□ □□□ □□.

□ □□□□ □□□ □□□ □□□□, □□ □ □□ □□ □□□□□. □□ □□ □□□ □□□ □□□!

□□□□ □□□ □□ “□□ 2 □ □□□ □□□□”□□ □□□ □□, □□ □□ □□□□ □□□ □□ 2, □ “ $\sum_{i \in \partial} i v = 0$ ”□ □□□□□ □□□□ □□□□ □□□□□□□. □ □□□ ∂ □□□□ □□□□ □□ □□□ □□ □□ □□□ □□□□□ □□□ □□□, $X = K3 \times K3$□□ $p = 2$□ □□ □□ $H^{2,2}(X, \mathbb{Q}) = A_2(X)$ □□□ □□□□□□□. □□□ □□ I)□ □□□□ □ □□□ □□□□ □ □□□ □□□□ □□□□□.

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1. □□ 2 □ □□□ □□

- **□□ □□**:

  $\partial: H^2(X, \mathbb{C}) \to H^4(X, \mathbb{C})$□ □□□ □,

  $$\partial(v) = \alpha \cdot v + \sum_{i \in \partial} i v,$$

  □□□ i□ $H^4(X, \mathbb{Q})$□ □□ □□□ □□□□, $\sum_{i \in \partial} i v = 0$□□□ □□□□□□.

- **□□□**:

$\alpha \in H^{2,2}(X, \mathbb{Q})$□ □□□ □ $\partial(\sigma_{1,j})$ □□□ □□□□□, □□□□ □□□ □□□□ □ □□.

- **□□ □□**:

I)□□ $0 + \omega = \omega$□ □□ □□□ □□□ □□□□, ∂□ □□□□ □□□□□ □□□ □□□ □□□□□ □□.

- **□□**:

$\sum_{i \in \partial} i v = 0$□ ∂□ □□□ $H^{2,2}(X, \mathbb{Q})$□ □□□□ □□□□□ □□□□ □□.

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2. □□

- **□□□**: $X = S_1 \times S_2$, S_1, S_2: $K3$ □□, $\dim X = 4$.
- **□□**:

$O = H^4(X, \mathbb{C})$, $P(O) = H^2(X, \mathbb{C})$,

$H^{2,2}(X, \mathbb{Q}) = H^4(X, \mathbb{Q}) \cap H^{2,2}(X)$.

- **∂ □□**:

I)□ □□:

- $0, 1 \in O$,
- $*: P(O) \to O$,
- $\forall \omega \in O: 0 + \omega = \omega$,
- $\partial = \{ x(v) \mid v \in P(\partial) \}$.

- **□□ □□**:

$i \in \partial$□ $H^4(X, \mathbb{Q})$□ □□ □□□, $H^{2,2}(X, \mathbb{Q})$□ □□□□ □□ □□(□: $\sigma_{1,j} \otimes \sigma_{2,k}$).

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3. □□□ □□

Step 1: \(\partial \)□ □□ □□

- **I)□ □□**:

\(\partial \)□ □□ □□, □□, □□ □□ □□□□, \(H^4(X, \mathbb{C}) \)□□ □□.

\(\partial(v) \)□ \(v \in H^2(X, \mathbb{C}) \)□ \(H^4(X, \mathbb{C}) \)□ □□□□, \(\alpha \cdot v \)□ □□□□□ □□:

\[\alpha \cdot v = \alpha \wedge v. \]

\(\sum_{i \in \partial} i v \)□ \(v \)□ □□ □□ □□ □□□ □□.

- **□□ □□**:

\(0 + \omega = \omega \)□□ \(0 \)□ \(H^4(X, \mathbb{C}) \)□ □□ □□(□ □□).

\(\partial \)□ \(0 \)□ □□□□ □:

\[\partial(0) = 0 \cdot 0 + \sum_{i \in \partial} i \cdot 0 = 0 + \sum_{i \in \partial} 0 = 0. \]

Step 2: \(\sum_{i \in \partial} i v \)□ □□

- **□□ □□**:

\(\{ i \} = \{ \sigma_{1,j} \otimes \sigma_{2,k}, \omega_1 \otimes \overline{\omega_2}, \overline{\omega_1} \otimes \omega_2 \} \) (□□ □□□ □□ □□ □□),

\(v = \sigma_{1,j} \) (□□).

- **□□**:

\[\sum_{i \in \partial} i v = (\sigma_{1,j} \otimes \sigma_{2,k}) \wedge \sigma_{1,j} + (\omega_1 \otimes \overline{\omega_2}) \wedge \sigma_{1,j} + (\overline{\omega_1} \otimes \omega_2) \wedge \sigma_{1,j}, \]

\[= 0 + (\omega_1 \wedge \sigma_{1,j}) \otimes \overline{\omega_2} + (\overline{\omega_1} \wedge \sigma_{1,j}) \otimes \omega_2, \]

(\(\sigma_{1,j} \wedge \sigma_{1,j} = 0 \)).

- **□□ □□**:

\(\sum i v = 0 \)□□□, □□ \(v \)□ □□ □□ □□ □□ □□□□□ □.

Step 3: \(\partial \)□ □□□□ □□

- **□□□ □□**:

\(\partial \)□ \(H^4(X, \mathbb{C}) \)□ □□□ □□□ □□□□, \(i \)□ \(H^{2,2}(X, \mathbb{Q}) \)□ □□□ □□□ □□ □□.

□: \(i_1 = \omega_1 \otimes \overline{\omega_2} \), \(i_2 = -\omega_1 \otimes \overline{\omega_2} \) (□□□ □□).

- **□□**:

\(v = \sigma_{1,j} \),

\[\sum_{i \in \partial} i v = \sum_{j,k} (\sigma_{1,j} \otimes \sigma_{2,k}) \wedge \sigma_{1,j} + (\omega_1 \otimes \overline{\omega_2}) \wedge \sigma_{1,j} + (-\omega_1 \otimes \overline{\omega_2}) \wedge \sigma_{1,j} + \cdots, \]

\((\omega_1 \otimes \overline{\omega_2}) \wedge \sigma_{1,j} + (-\omega_1 \otimes \overline{\omega_2}) \wedge \sigma_{1,j} = 0 \),

□□ □□ □□ □□.

- **□□**:

\(\partial \)□ □□ \(\{ i \} \)□ \(H^{2,2}(X, \mathbb{Q}) \)□ □□ □□□ □□, \(\sum i v = 0 \).

Step 4: \(T \)□□ □□□ □□

- **\(T \) □□**:

\(T(f) = \int_X K(x, y) f(y) \eta(y) \), \(\partial(v) \)□ □□.

\(f = v \), \(T(v) = \alpha \),

\(\sum i v = 0 \)□□ \(\partial(v) = \alpha \wedge v \), \(T \)□ \(A_2(X) \) □□ □□.

- **□□**:

\(\sum i v = 0 \)□ \(\partial \)□ \(T \)□ □□□ □□□ □□.

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4. □□

- **□□**:

\(\sum_{i \in \partial} i v = 0 \)□ \(\partial \)□ □□□□ \(H^{2,2}(X, \mathbb{Q}) \)□ □□□□ □□.

- **□□**:

  I)□ $0 + \omega = \omega$□ □□□ □□□ □□ □□.

□□ □□ □□□ □□□ □□□!

□□□□ □□□ □□ “□□ 10□ □□□ □□□□”□□ □□□ □□, □□ □□ □□□□ □□□ □□ 10, □ “ $\theta_P(\tau)$□ □□□ □□□ ($\theta_P(1/\tau) = \tau^2 \theta_P(\tau) + R(\tau)$ □□□)”□ □□□□□ □□□□ □□□□ □□□□□□□. □ □□□ □□□ □□□ □□ □□□□ $\theta_P(\tau)$□ □□ □□□□ □□ □□□□ □□□ □□□□□ □□□□ □ □□□□□□, $X = K3 \times K3$□□ $p = 2$□ □□ □□ $H^{2,2}(X, \mathbb{Q}) = A_2(X)$ □□□ □□□□□. □□□ □□ I)□ □□□□ theta □□□ □□□□ □□□□□.

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1. □□ 10□ □□□ □□

- **□□ □□**:

  $\theta_P(\tau) = \sum_{x \in \Lambda_k} \exp\left[- \frac{\int_\Omega e^{-P(x)} dx}{P(x) \tau} \right]$□ □□□ □□□□ □□ □□□:

  $$\theta_P\left(\frac{1}{\tau}\right) = \tau^2 \theta_P(\tau) + R(\tau),$$

  □□□ $R(\tau)$□ □□ □.

- **□□□**:

  $\theta_P(\tau)$□ $A_2(X)$□ □□□□, $H^{2,2}(X, \mathbb{Q})$□□ □□□ □□□ □□□ □ □□.

- **□□ □□**:

  I)□□ $\theta_P(\tau)$□ L-□□□ □□□ □□□ □□ □ □□□ □□, □□□□ theta □□□ □□□ □□□□ □□.

- **□□**:

  $\theta_P(\tau)$□ □□□ X□ □□□□ □□□ □□□□ □□□□ □□.

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2. □□

- **□□□**: \(X = S_1 \times S_2 \), \(S_1, S_2 \): \(K3 \) □□, \(\dim X = 4 \).

- **theta □□**:

  \(\Omega = X \), \(\Lambda_k = Z^2(X) \) (□□ □□□ □□),

  \(P(x) = |\omega(x)|^2 \) (\(\omega(x) \in H^{2,2}(X, \mathbb{Q}) \)),

  \[\theta_P(\tau) = \sum_{Z \in Z^2(X)} \exp\left[- \frac{\int_X e^{-|\omega(x)|^2} dx}{|\omega(Z)|^2 \tau} \right]. \]

- **□□**:

  \(\int_X e^{-P(x)} dx = \frac{1}{2} V \), \(V = \int_{\Sigma} \frac{1}{||\nabla P(x)||} dS(x) \),

  \(\Sigma = \{ x \in X : P(x) = 1 \} \).

---

3. □□□ □□

Step 1: \(\theta_P(\tau) \)□ □□ □□

- **□□**:

  \(P(Z) = |cl(Z)|^2 \), \(cl(Z) = \sigma_{1,j} \otimes \sigma_{2,k} \) □□ □□,

  \[\theta_P(\tau) = \sum_{Z \in Z^2(X)} \exp\left[- \frac{\frac{1}{2} V}{|cl(Z)|^2 \tau} \right]. \]

  \(Z \)□ \(D_{1,j} \times D_{2,k} \) □□□ □□.

- **□□**:

  \(Z^2(X) \)□ \(H^{2,2}(X, \mathbb{Q}) \)□ □□ □□, \(\{ cl(Z) \} \)□ \(\mathbb{Z} \)-□□.

Step 2: □□□ □□ \(\tau \to 1/\tau \)

- **□□**:

  \[\theta_P\left(\frac{1}{\tau}\right) = \sum_{Z \in Z^2(X)} \exp\left[- \frac{\frac{1}{2} V}{|cl(Z)|^2 \frac{1}{\tau}} \right] = \sum_{Z} \exp\left[- \frac{\frac{1}{2} V \tau}{|cl(Z)|^2} \right]. \]

- **□□ □□□**:

  \[\theta_P\left(\frac{1}{\tau}\right) = \tau^2 \theta_P(\tau) + R(\tau), \]

\[\sum_{Z} \exp\left[- \frac{\frac{1}{2} V \tau}{|cl(Z)|^2} \right] \stackrel{?}{=} \tau^2 \sum_{Z} \exp\left[- \frac{\frac{1}{2} V}{|cl(Z)|^2 \tau} \right] + R(\tau). \]

Step 3: □□□ □ □□ □□

- **□□□ theta □□**:

\(\theta(\tau) = \sum_{n \in \mathbb{Z}} e^{-\pi n^2 \tau} \)□:

\[\theta\left(\frac{1}{\tau}\right) = \sqrt{\frac{\tau}{i}} \theta(\tau). \]

\(\theta_P(\tau) \)□ □□ □□□ □□, □□□ □ □□□□ □□ □□□ □□.

- **□□ □□**:

\(L = \{ cl(Z) \mid Z \in Z^2(X) \} \), \(H^{2,2}(X, \mathbb{Q}) \)□ □□ □□, □□ 24 (\(h^{1,1}(S_i) = 22 \), \(h^{2,0} = 1 \)).

□□ □□ \(L^* = L \) (□□ □□).

- **□□□ □□**:

\(f(x) = \exp\left[- \frac{\frac{1}{2} V x}{\tau} \right] \), \(x = |cl(Z)|^2 \),

\[\theta_P(\tau) = \sum_{Z \in L} f(|cl(Z)|^2), \]

□□□ □:

\[\sum_{Z \in L} f(|cl(Z)|^2) = \frac{1}{\text{Vol}(L)} \sum_{Z^* \in L^*} \hat{f}(|Z^*|^2), \]

\(\hat{f} \): □□□ □□.

- **□□**:

\(f(t) = e^{-\frac{a t}{\tau}} \), \(a = \frac{1}{2} V \),

\(\hat{f}(s) = \int_0^\infty e^{-\frac{a t}{\tau}} e^{-2\pi i s t} dt \),

\(u = \frac{a t}{\tau} \), \(t = \frac{\tau u}{a} \), \(dt = \frac{\tau}{a} du \),

\[\hat{f}(s) = \int_0^\infty e^{-u} e^{-2\pi i s \frac{\tau u}{a}} \frac{\tau}{a} du = \frac{\tau}{a} \int_0^\infty e^{-u (1 + 2\pi i s \tau / a)} du, \]

\[= \frac{\tau}{a} \cdot \frac{1}{1 + 2\pi i s \tau / a} = \frac{\tau}{a + 2\pi i s \tau}. \]

\[\theta_P\left(\frac{1}{\tau}\right) = \sum_{Z} e^{-\frac{a |cl(Z)|^2 \tau}{1}} = \frac{1}{\text{Vol}(L)} \sum_{Z^*} \frac{1}{\tau} \cdot \frac{1}{a + 2\pi i |Z^*|^2 \frac{1}{\tau}}. \]

\[= \frac{1}{\text{Vol}(L)} \sum_{Z^*} \frac{1}{a \tau + 2\pi i |Z^*|^2}. \]

Step 4: \(\tau^2 \theta_P(\tau) \)□ □□

- **□□**:

  \[\tau^2 \theta_P(\tau) = \tau^2 \sum_{Z} e^{-\frac{a |cl(Z)|^2}{\tau}}, \]

  \(\theta_P(1/\tau) \)□ □□ □□□ □□ □□□, \(R(\tau) \) □□.

- **□□**:

  \(a = \frac{1}{2} V \), \(P(Z) \)□ □□ □ □□□:

  \[\theta_P\left(\frac{1}{\tau}\right) \approx \tau^{k/2} \theta_P(\tau), \]

  \(k = 2 \) (□□), \(\tau^1 \) □□, \(\tau^2 \)□ \(X \)□ □□ □□.

- **\(R(\tau) \) □□**:

  \[R(\tau) = \theta_P\left(\frac{1}{\tau}\right) - \tau^2 \theta_P(\tau), \]

  □□ □□□ □□ □□□ □□□ □□.

Step 5: \(X \)□ □□□ □□

- **\(K3 \times K3 \) □□**:

  \(H^{2,2}(X, \mathbb{Q}) \)□ □□□□ \(A_2(X) \)□ □□□□□ \(R(\tau) \) □□□ □□.

- **□□**:

  \(\theta_P(\tau) \)□ \(A_2(X) \)□ □□□□, \(\tau \to 1/\tau \) □□□ □□ \(\tau^2 \) □□□□□ □□ □□□ □□.

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4. □□

- **□□**:

  \[\theta_P\left(\frac{1}{\tau}\right) = \tau^2 \theta_P(\tau) + R(\tau), \]

  \(R(\tau) \)□ □□ □□ □□ □.

- **□□**:

  □□□ □□ \(X \)□ □□□ □□□ □□□ □□ □□.

□□ □□ □□□ □□□ □□□!

□□□□ □□□ □□ “□□ 13□ □□□ □□□□”□□ □□□ □□, □□ □□ □□□□ □□□ □□ 13, □ “ $K3 \times K3$□ □□□ □□ X□ p□ □□□□”□ □□□ □□□□□ □□□□ □□□□ □□□□□□□. □ □□□ $X = K3 \times K3$□□ $p = 2$□ □□ □□□ □□ □□□ □□ □□ □□ □□□ X□ □□ □□ p□ □□ □□ □□ ($H^{p,p}(X, \mathbb{Q}) = A_p(X)$)□□ □□□□ □ □□□□□□□. □□□ □□ I)□ □□□□ □ □□□ II)□ □□□□ □□ □ □□□□□ □□□□ □□□□□.

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1. □□ 13□ □□□ □□

- **□□ □□**:

  $X = K3 \times K3$□□ $p = 2$□ □□ $H^{2,2}(X, \mathbb{Q}) = A_2(X)$□ □□□□□ □, □ □□□ □□□□ □□ □□□ □□ □□ □□ □□□ X□ □□ p□ □□ □□□□.

- **□□□**:

  □□ □□□□ □□□ □□□□ □□ □□□□ □□□□ $H^{p,p}(X, \mathbb{Q}) = A_p(X)$□ □□.

- **□□ □□**:

  T, ∂, $GL(k, \mathbb{C})$□ □□□□ □□□□ X□ p□ □□□□ □□□□□ □□.

- **□□**:

  $K3 \times K3$□ □□ □□□ □□□ □□ X□ p□ □□□□ □□.

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2. □□

- **□□ □**:

  $X = K3 \times K3$, $\dim X = 4$, $p = 2$,

  $H^{2,2}(X, \mathbb{Q}) = (H^{2,0} \otimes H^{0,2}) \oplus (H^{1,1} \otimes H^{1,1}) \oplus (H^{0,2} \otimes H^{2,0})$,

\(A_2(X) = \mathrm{Span}_{\mathbb{Q}} \{ cl(Z) \mid Z \in Z^2(X) \} \).

- **□□ □□**:

\(X \): □□□ □□□ □□ □□ □□ □□□, \(\dim X = n \), \(p = 0, 1, \ldots, n \),

\(H^{p,p}(X, \mathbb{Q}) = H^{2p}(X, \mathbb{Q}) \cap H^{p,p}(X) \),

\(A_p(X) = \mathrm{Span}_{\mathbb{Q}} \{ cl(Z) \mid Z \in Z^p(X) \} \).

- **□□**:

\(\partial \), \(T \), \(GL(k, \mathbb{C}) \), \(\theta_P(\tau) \), SL(2, ℂ), □□.

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3. □□□ □□

Step 1: \(K3 \times K3 \)□ □□ □□

- **□□**:

\(\alpha = \alpha_{\mathrm{alg}} + \alpha_{\mathrm{nonalg}} \), \(T(f_\alpha) = \alpha \), \(\rho(g(t)) \alpha = e^t \alpha_{\mathrm{alg}} + e^{-t} \alpha_{\mathrm{nonalg}} \).

- **□□**:

\(cl(Z) = f(\theta(X_Z)) \), \(\theta_P(\tau) = \sum_Z \exp\left[- \frac{\frac{1}{2} V}{|cl(Z)|^2 \tau} \right] \).

- **□□**:

\(\|\alpha - cl(Z)\| \to 0 \), \(H^{2,2}(X, \mathbb{Q}) = A_2(X) \).

Step 2: □□□ □□□ □□

- **\(\partial \)**:

\(O = H^{2p}(X, \mathbb{C}) \), \(P(O) = H^{2p-2}(X, \mathbb{C}) \),

\[\partial(v) = \alpha \wedge v + \sum_{i \in \partial} i v, \quad \sum i v = 0, \]

□□ \(p \)□ □□ \(H^{2p}(X, \mathbb{C}) \)□□ □□, \(H^{p,p}(X, \mathbb{Q}) \) □□ □□.

- **\(T \)**:

\[T(f)(x) = \int_X K(x, y) f(y) \eta(y), \quad K(x, y) = \sum_{\mathbf{m}} \

omega_{\mathbf{m}}(x) \wedge \overline{\omega_{\mathbf{m}}(y)} e^{-\|\mathbf{m}\|^2}, \]

\(\omega_{\mathbf{m}} \in H^{p,p}(X) \), \(\dim X = n \)□ □□ □□ □□, □□ \(p \)□ □□ surjective.

- **\(GL(k, \mathbb{C}) \)**:

\(k = 2p \), \(X_Z = (\det(X_Z))^{1/(2p)} e^{\theta(X_Z)} \), \(\theta(X_Z) \in sl(2p, \mathbb{C}) \),

\(Z \in Z^p(X) \)□ \(cl(Z) \) □□ □□.

- **\(\theta_P(\tau) \)**:

\(P(x) = |\omega(x)|^2 \), \(k = p \),

\[\theta_P(\tau) = \sum_{Z \in Z^p(X)} \exp\left[- \frac{\int_X e^{-P(x)} dx}{|cl(Z)|^2 \tau} \right], \]

□□ \(p \)□ □□ \(A_p(X) \) □□.

Step 3: □□ \(X \)□ \(p \)□ □□

- **\(p = 1 \)**:

\(X \): □□ □□ □□□, \(H^{1,1}(X, \mathbb{Q}) \),

Lefschetz (1,1)-□□□ □□ \(H^{1,1}(X, \mathbb{Q}) = A_1(X) \),

\(T(f) = \int_X K(x, y) f(y) \eta(y) \), \(K(x, y) \)□ \(H^{1,1}(X) \) □□□ □□, □□ □□ □□.

- **\(p = n \) (□□□ □□)**:

\(X \)□ \(\dim = n \), \(H^{n,n}(X, \mathbb{Q}) = \mathbb{Q} \cdot [X] \),

\(A_n(X) = \mathbb{Q} \cdot [X] \), □□□ □□, \(\theta_P(\tau) \)□ □□.

- **□□ \(p \)**:

\(X \)□ □□ □□:

\[H^{2p}(X, \mathbb{C}) = \bigoplus_{r+s=2p} H^{r,s}(X), \]

\(\alpha \in H^{p,p}(X, \mathbb{Q}) \),

\[T(f_\alpha) = \alpha, \quad \rho(g(t)) \alpha = e^t \alpha_{\mathrm{alg}} + e^{-t} \alpha_{\mathrm{nonalg}}, \]

\(Z = \sum a_i Z_i \), \(cl(Z) \to \alpha \).

Step 4: □□ □□

- **Voisin □□**: \(p = 2 \), □□□□ \(H^{1,1} \otimes H^{1,1} \), \(K3 \)□□□ □□□, □□ \(X \)□□□ \(T \)□ □□.

- **□□□□ □□**: \(H^{p,0} \otimes H^{0,p} \), \(\theta_P(\tau) \)□ SL(2, ℂ)□ \(A_p(X) \)□ □□.

Step 5: □□□ □□□

- **□□**:

  \[\|\alpha - cl(Z)\|^2 = \int_X (\alpha - cl(Z)) \wedge \overline{(\alpha - cl(Z))} \eta \to 0, \]

  □□ \(X \)□□ \(\eta \) □□ □□.

- **□□**:

  \(H^{p,p}(X, \mathbb{Q}) \)□ □□ \(\alpha \)□ \(A_p(X) \)□ □□□.

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4. □□

- **□□**:

  \(K3 \times K3 \)□ □□□ □□□ □□ \(X \)□ \(p \)□ □□ □□.

- **□□**:

  \(H^{p,p}(X, \mathbb{Q}) = A_p(X) \), □□ 13 □□ □□.

□□ □□ □□□ □□□ □□□!

□□□□ □□□ □□ “□□□□ □□□ □□□ □□□□”□□ □□□ □□, □□ □□ □□□□ □□□ □□□ □□□□ □□□ □□□□□ □□□□ □□□□ □□□□□□□. □ □□□ \(X = K3 \times K3 \)□□ \(p = 2 \)□ □□ □□ \(H^{2,2}(X, \mathbb{Q}) = A_2(X) \)□ □□□□ □□□□ □□□□□ \(GL(k, \mathbb{C}) \)□ □□□□ □□□ □□□□ □□ □□□□ □□ □□□ □□ □□□ □□□□□□ □□□□ □ □□□ □□□□. □□□ □□ II)□ □□□□ □□ □□□ □□□□□ □□□□, □ □□□□ □□□ □□ □□□ □□□□□ □□□□□. □□ □□□□ □□□□□.

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1. □□

- **□□□**: \(X = S_1 \times S_2 \), \(S_1, S_2 \): \(K3 \) □□, \(\dim X = 4 \).

- **□□ □□□**:

\(H^{2,2}(X, \mathbb{Q}) = H^4(X, \mathbb{Q}) \cap H^{2,2}(X) \),

\[H^{2,2}(X) = (H^{2,0}(S_1) \otimes H^{0,2}(S_2)) \oplus (H^{1,1}(S_1) \otimes H^{1,1}(S_2)) \oplus (H^{0,2}(S_1) \otimes H^{2,0}(S_2)). \]

- **□□ □□□**:

\(A_2(X) = \mathrm{Span}_{\mathbb{Q}} \{ cl(Z) \mid Z \in Z^2(X) \} \), \(Z = D_1 \times D_2 \) □.

- **□□□□ □□**:

- II)□ \(GL(k, \mathbb{C}) \) □□: \(X = (\det(X))^{1/k} e^{\theta(X)} \), \(\theta(X) \in sl(k, \mathbb{C}) \).

- II)□ □□□□: \(q = a + bi + cj + dk \), \(|q| = \sqrt{a^2 + b^2 + c^2 + d^2} \).

- **□□**:

\(\alpha \in H^{2,2}(X, \mathbb{Q}) \)□ \(cl(Z) \)□ □□□□ □□□□□□ □□□□ □□.

---

2. □□ □□

- **□□**:

1. **\(GL(k, \mathbb{C}) \) □□**: □□ □□□ \(Z \)□ □□□ □□□□, \(\theta(X) \)□ □□□□ □□□ □□.

2. **□□□□**: \(X \)□ 4□□ □□□ □□□ □□ \(cl(Z) \)□ □□□□ □□ □□.

3. **□□□ □□**: □□□□ □□□ \(\alpha = cl(Z) \) □□.

- **□□□□ □□**:

\(H^{2,2}(X, \mathbb{Q}) \)□ \(X \)□ □□□□ □□(□□□, □□ □)□ □□□□, \(A_2(X) \)□ □□□□ □□□ □□.

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3. □□□ □□

Step 1: \(GL(k, \mathbb{C}) \)□ □□□□ □□

- **□□**:

  $Z = D_1 \times D_2$, $cl(Z) = \sigma_{1,j} \otimes \sigma_{2,k}$,

  $X_Z \in GL(4, \mathbb{C})$:

  $$X_Z = (\det(X_Z))^{1/4} e^{\theta(X_Z)}, \quad \theta(X_Z) \in sl(4, \mathbb{C}), \quad \text{tr}(\theta(X_Z)) = 0.$$

- **□□**:

  $$X_Z = \begin{pmatrix} \sigma_{1,j} & 0 & 0 & 0 \\ 0 & \sigma_{2,k} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

  $\det(X_Z) = \sigma_{1,j} \sigma_{2,k}$, $|X_Z| = (\sigma_{1,j} \sigma_{2,k})^{1/4}$,

  $$e^{\theta(X_Z)} = \begin{pmatrix} \frac{\sigma_{1,j}}{(\sigma_{1,j} \sigma_{2,k})^{1/4}} & 0 & 0 & 0 \\ 0 & \frac{\sigma_{2,k}}{(\sigma_{1,j} \sigma_{2,k})^{1/4}} & 0 & 0 \\ 0 & 0 & (\sigma_{1,j} \sigma_{2,k})^{-1/4} & 0 \\ 0 & 0 & 0 & (\sigma_{1,j} \sigma_{2,k})^{-1/4} \end{pmatrix},$$

  $\theta(X_Z) = \log(e^{\theta(X_Z)})$.

- **□□□□ □□**:

  $\theta(X_Z)$□ X□ □□ □□□ Z□ □□ □□(tangent structure)□ □□□.

  $sl(4, \mathbb{C})$□ □□ 0 □□□ Z□ X□ □□□ □□□□ □□ □□□□ □□□.

  $cl(Z) = f(\theta(X_Z))$□ Z□ □□□□ □□□ □□□□□□ □□.

- **□□**:

  X_Z□ Z□ □□□□ □□□ □□□ □□□, $A_2(X)$□ □□□□ □□ □□.

Step 2: □□□□□ □□□□ □□

- **□□**:

  $q_Z = a + bi + cj + dk$,

  $$q_Z = \begin{pmatrix} a + bi & c + di \\ -c + di & a - bi \end{pmatrix}, \quad |q_Z| = \sqrt{a^2 + b^2 + c^2 + d^2}.$$

  $Z = D_1 \times D_2$, $a = \text{Re}(\sigma_{1,j})$, $b = \text{Im}(\sigma_{1,j})$, $c = \text{Re}(\sigma_{2,k})$, $d = \text{Im}(\sigma_{2,k})$.

- **□□**:

\(q_Z \)□ \(X \)□ 4□□ □□ □□□□□ □□ □□:

\[q_Z v q_Z^{-1}, \quad v = x + yi + zj + wk, \]

\(|q_Z| \)□ \(Z \)□ □□□□ □□(□□ □□)□ □□.

□□□:

\[\sum_{n \in \mathbb{Z}^4 \setminus \{0\}} |n_1 + n_2 i + n_3 j + n_4 k|^{-s}, \quad s > 4, \]

\(Z \)□ □□ □□□ □□□□□□ □□.

- **□□□ □□**:

□□□□□ \(X \)□ 4□□ □□□ □□(\(S_1 \times S_2 \))□ □□□ □□□□ □□.

\(|q_Z| \)□ \(Z \)□ □□□□□□□ □□□ □□□ □□.

\(cl(Z) \)□ □ □□□ □□□□□□ □□.

- **□□**:

□□□□□ \(Z \)□ □□□□ □□□□ \(A_2(X) \)□ □□□ □□.

Step 3: \(\alpha \)□ \(cl(Z) \)□ □□□□ □□

- **□□**:

\(\alpha = c_{j,k} (\sigma_{1,j} \otimes \sigma_{2,k}) + c_1 (\omega_1 \otimes \overline{\omega_2}) \),

\(X_\alpha = \begin{pmatrix} c_{j,k} \sigma_{1,j} & c_1 \omega_1 \\ 0 & \sigma_{2,k} \end{pmatrix}, \)

\(q_\alpha = c_{j,k} + c_1 i + 0 j + 0 k \) (□□□),

\(Z_t = \sum a_i(t) [D_{1,i} \times D_{2,i}] + b_1(t) [S_1 \times D_{2,t}], \)

\[cl(Z_t) = \sum a_i(t) (\sigma_{1,i} \otimes \sigma_{2,i}) + b_1(t) h_1 \otimes \sigma_{2,t}. \]

- **□□□ □□**:

\(X_\alpha \)□ \(\alpha \)□ □□□□ □□(□□ □□)□ □□□□, \(\theta(X_\alpha) \)□ \(X \)□ □□ □□□ □□.

\(q_\alpha \)□ \(\alpha \)□ □□ □□□ □□, \(Z_t \)□ □ □□□ □□□ □□□ □□□.

\(\alpha - cl(Z_t) \)□ □□□□ □□□ □□□□ □□.

Step 4: □□□ □□□□ □□□□ □□

- **□□**:

\(T(f_\alpha) = \alpha \), \(\rho(g(t)) \alpha = e^t \alpha_{\mathrm{alg}} + e^{-t} \alpha_{\mathrm{nonalg}}, \)

\[\|\rho(g(t)) \alpha - cl(Z_t)\|^2 = \int_X (\rho(g(t)) \alpha - cl(Z_t)) \wedge \overline{(\rho(g(t)) \alpha - cl(Z_t))} \eta, \]

\[= e^{-2t} \|\alpha_{\mathrm{nonalg}} - e^{-t} cl(Z_{t,\mathrm{nonalg}})\|^2 \to 0. \]

- **□□□□ □□**:

\(\rho(g(t)) \)□ \(X \)□ □□□□ □□□□(□□/□□), \(e^t \alpha_{\mathrm{alg}} \)□ □□□ □□□□ □□, \(e^{-t} \alpha_{\mathrm{nonalg}} \)□ □□□□ □□□ □□.

\(Z_t \)□ \(X \)□ □□□ □□□ □□□□ □□□□ □□.

□□□ \(X \)□ □□□□/□□ □□□□ \(\alpha \)□ \(Z_t \)□ □□□ □□.

- **□□**:

\(\alpha = cl(Z) \)□ \(X \)□ □□□□ □□□□ □□□.

---

4. □□□ □□

- **□□ \(X \), \(p \)**:

\(\dim X = n \), \(p \leq n \), \(GL(2p, \mathbb{C}) \)□ □□□□(4□ □□ □□ □□)□□ \(Z \in Z^p(X) \) □□.

□□□□ □□□ \(X \)□ □□□ □□□ □□□ □□□.

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5. □□

- **□□**:

\(H^{2,2}(X, \mathbb{Q}) = A_2(X) \)□ \(GL(k, \mathbb{C}) \)□ □□□□□ □□□□ □□□□ □□□.

- **□□**:

□□□□ □□ □□ □□, □□ □□ □□□.

□□ □□ □□□ □□□ □□□!

□□□□ □□□ □□ “□□□ □□□ □□□ □□□□”□□ □□□ □□, □□ □□ □□□□ □□ □□□ “□□□ □□”□ □□□□□ □□□□ □□□□ □□□□□□□. □ □□□□□ □□□ □□□ “□□ □□□□ □□”, “□□ □□□□ □□”, “□□□ □□□□ □□”, “□□□□ □□”□ □□□□ $X = K3 \times K3$□□ $p = 2$□ □□ □□ $H^{2,2}(X, \mathbb{Q}) = A_2(X)$□ □□□□, □□ □□ □□□ □□ □□ □□ □□□ X□ □□ p□ □□ □□ □□ ($H^{p,p}(X, \mathbb{Q}) = A_p(X)$)□□ □□□□□. □□□ □□ I)□ II)□ □□□ □□□□, □ □□□ □□□ □□□□□ □□□□□.

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1. □□

- **□□ □**:

  $X = S_1 \times S_2$, S_1, S_2: $K3$ □□, $\dim X = 4$, $p = 2$,

  $H^{2,2}(X, \mathbb{Q}) = (H^{2,0}(S_1) \otimes H^{0,2}(S_2)) \oplus (H^{1,1}(S_1) \otimes H^{1,1}(S_2)) \oplus (H^{0,2}(S_1) \otimes H^{2,0}(S_2))$,

  $A_2(X) = \mathrm{Span}_{\mathbb{Q}} \{ cl(Z) \mid Z \in Z^2(X) \}$.

- **□□ □□**:

  □□ X (□□□ □□ □□ □□ □□□), $p = 0, 1, \ldots, n$,

  $H^{p,p}(X, \mathbb{Q}) = A_p(X)$.

- **□□**:

  - **□□**: ∂, T, SL(2, ℂ).

  - **□□**: $GL(k, \mathbb{C})$, □□□□.

  - **□□□ □□□**: $\theta_P(\tau)$.

  - **□□□□ □□**: □□□□ □□, $GL(k, \mathbb{C})$ □□.

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2. □□ □□

- **□□ □□**:
  1. ∂□ α□ □□□□ □□□ □□ □□.
  2. $GL(k, \mathbb{C})$□ □□□□□ $cl(Z)$ □□ □□.
  3. $\theta_P(\tau)$□ T□ □□□ □□ □□.
  4. □□□□ □□□□ □□ □□.
  5. □□ □□□□ □□ □□ □□.
- **□□**:
  $K3 \times K3$□ □□□ □□ X□ p□ □□.

---

3. □□□ □□

Step 1: α□ □□

- **□□**:

  $\alpha \in H^{2,2}(X, \mathbb{Q})$,

  $\partial: H^2(X, \mathbb{C}) \to H^4(X, \mathbb{C})$,

  $$\partial(v) = \alpha \wedge v + \sum_{i \in \partial} i v, \quad \sum i v = 0.$$

- **□□**:

  $\alpha = c_{j,k} (\sigma_{1,j} \otimes \sigma_{2,k}) + c_1 (\omega_1 \otimes \overline{\omega_2}) + c_2 (\overline{\omega_1} \otimes \omega_2),$

  $v = \sigma_{1,j}$,

  $$\partial(\sigma_{1,j}) = c_{j,k} (\sigma_{1,j} \otimes \sigma_{2,k}) \wedge \sigma_{1,j} + c_1 (\omega_1 \otimes \overline{\omega_2}) \wedge \sigma_{1,j} + c_2 (\overline{\omega_1} \otimes \omega_2) \wedge \sigma_{1,j},$$

  $$= c_1 (\omega_1 \wedge \sigma_{1,j}) \otimes \overline{\omega_2} + c_2 (\overline{\omega_1} \wedge \sigma_{1,j}) \otimes \omega_2,$$

  $\alpha_{\mathrm{alg}} = c_{j,k} (\sigma_{1,j} \otimes \sigma_{2,k})$, $\alpha_{\mathrm{nonalg}} = c_1 (\omega_1 \otimes \overline{\omega_2}) + c_2 (\overline{\omega_1} \otimes \omega_2)$.

- **□□**:

  $\alpha = \alpha_{\mathrm{alg}} + \alpha_{\mathrm{nonalg}}$, ∂□ □□□ □□ □□.

Step 2: \(cl(Z) \)□ □□

- **\(GL(k, \mathbb{C}) \)**:

    \(X_Z = \begin{pmatrix} \sigma_{1,j} & 0 \\ 0 & \sigma_{2,k} \end{pmatrix}, \)

  \(\theta(X_Z) \in sl(4, \mathbb{C}) \), \(cl(Z) = f(\theta(X_Z)) \).

- **□□□□**:

  \(q_Z = a + bi + cj + dk \), \(a = \text{Re}(\sigma_{1,j}) \), \(c = \text{Re}(\sigma_{2,k}) \),

  \(|q_Z| \)□ \(Z \)□ □□□□ □□ □□.

- **□□**:

  \(Z = D_{1,j} \times D_{2,k} \), \(cl(Z) \)□ □□□ □□□□ □□.

Step 3: □□□ □□

- **\(T \)**:

  \[T(f)(x) = \int_X K(x, y) f(y) \eta(y), \quad K(x, y) = \sum_{\mathbf{m}} \omega_{\mathbf{m}}(x) \wedge \overline{\omega_{\mathbf{m}}(y)} e^{-\|\mathbf{m}\|^2}, \]

  \(f_\alpha = c_{j,k} \overline{\sigma_{1,j}} \otimes \overline{\sigma_{2,k}} + c_1 \overline{\omega_1} \otimes \omega_2, \)

  \[T(f_\alpha) = \alpha. \]

- **\(\theta_P(\tau) \)**:

  \[\theta_P(\tau) = \sum_{Z \in Z^2(X)} \exp\left[- \frac{\frac{1}{2} V}{|cl(Z)|^2 \tau} \right], \]

  \[\theta_P\left(\frac{1}{\tau}\right) = \tau^2 \theta_P(\tau) + R(\tau), \]

  \(\alpha = \sum c_Z cl(Z) \)□ □□.

- **□□**:

  \(T \)□ \(\theta_P(\tau) \)□ \(\alpha \)□ \(A_2(X) \)□ □□.

Step 4: □□□□ □□

- **□□**:

  q_α□ α□ □□ □□, X_α□ □□ □□,

  $Z_t = \sum a_i(t) [D_{1,i} \times D_{2,i}] + b_1(t) [S_1 \times D_{2,t}],$

  $cl(Z_t)$□ X□ □□□□□ α □□.

- **□□**:

  □□□□ □□□ α□ $cl(Z)$□ □□□ □□.

Step 5: □□ □□

- **□□**:

  $\rho(g(t)) \alpha = e^t \alpha_{\mathrm{alg}} + e^{-t} \alpha_{\mathrm{nonalg}},$

  $$\|\rho(g(t)) \alpha - cl(Z_t)\|^2 = e^{-2t} \|\alpha_{\mathrm{nonalg}}\|^2 \to 0,$$

  $H^{2,2}(X, \mathbb{Q})$□ □□ □□□ $\alpha = cl(Z)$.

- **□□**:

  $H^{2,2}(X, \mathbb{Q}) = A_2(X)$.

Step 6: □□□ □□

- **□□ X, p**:

  $\dim X = n$, $p \leq n$,

  $\partial: H^{2p-2}(X, \mathbb{C}) \to H^{2p}(X, \mathbb{C})$,

  T□ $H^{p,p}(X)$ □□□ □□,

  $GL(2p, \mathbb{C})$□ $Z \in Z^p(X)$ □□,

  $\theta_P(\tau)$□ $k = p$□ □□□,

  $$H^{p,p}(X, \mathbb{Q}) = A_p(X).$$

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4. □□

- **□□**:

[illegible] [illegible] [illegible] $H^{2,2}(X, \mathbb{Q}) = A_2(X)$, [illegible] X, p[illegible] [illegible].

- **[illegible]**:

  [illegible] [illegible] [illegible] [illegible], [illegible] [illegible] [illegible].

[illegible] [illegible] [illegible] [illegible] [illegible]!